Thermonuclear fusion occurs at high temperature.

- For thermonuclear fusion, the reaction rate is given by a Maxwellian averaged cross section:

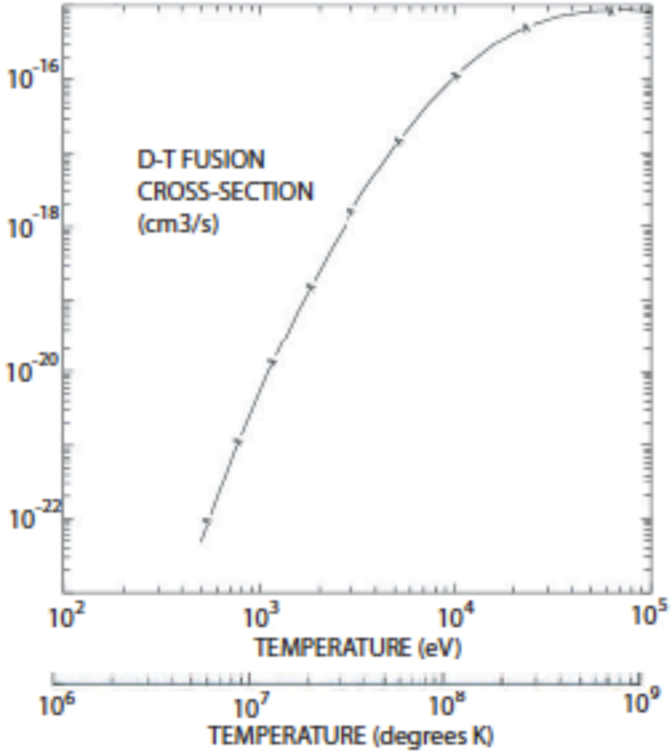
$$\frac{dn_1}{dt} = \frac{dn_2}{dt} = -\overline{\sigma v} n_1 n_2, \qquad n_1, n_2 = \#\ \text{particles/cm}^3$$

- D-T is the preferred fuel because of its higher cross-section

$$n_D = n_T = \frac{n_i}{2}, \qquad \frac{dn_i}{dt} = -\frac{\overline{\sigma v} n_i^2}{2}, \qquad n_i = \#\ \text{ions/cm}^3$$

- To reach a value of even 1% of peak, a temperature of 4 keV (50,000,000 deg. K) must be reached and sustained.

- A temperatures above ~ 2 eV, D-T is a plasma, i.e., a fully ionized gas of nuclei and electrons.



The rate equation can be integrated to give expressions for the number density and fusion energy produced as functions of time.

- **Number density:** $\frac{dn_i}{n_i^2} = -\frac{\overline{\sigma v}}{2} dt, \qquad \frac{\overline{\sigma v}}{2} t = \frac{1}{n_i} - \frac{1}{n_o}, \qquad n_i = \frac{n_o}{1 + \frac{\overline{\sigma v}}{2} n_o t}$

- **Energy produced:** $E_{FUS} = \varepsilon_{FUS} \frac{n_o - n_i}{2} = \varepsilon_{FUS} n_o^2 t \frac{\overline{\sigma v}}{4} \frac{1}{1 + \frac{\overline{\sigma v}}{2} n_o t}, \qquad \varepsilon_{FUS}$ **=17.6 Mev**

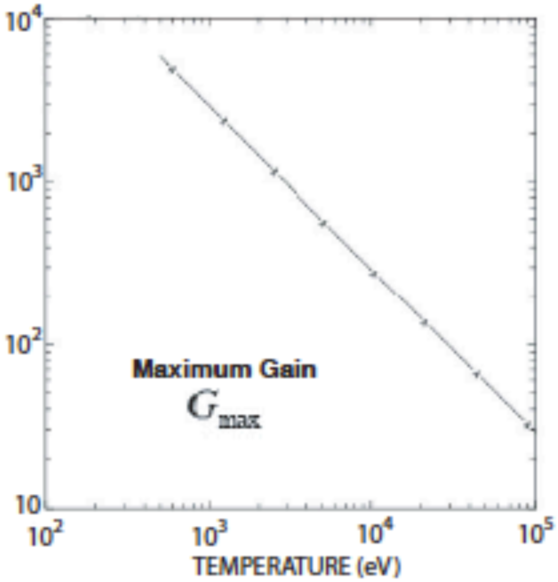
$$\dot{Q}_{FUS} = \frac{dE_{FUS}}{dt} = -\frac{\varepsilon_{FUS}}{2} \frac{dn_i}{dt} = \varepsilon_{FUS} n_i^2 \frac{\overline{\sigma v}}{4}$$

- **Initial plasma energy:** $3 n_o T_o$

- **Gain:**

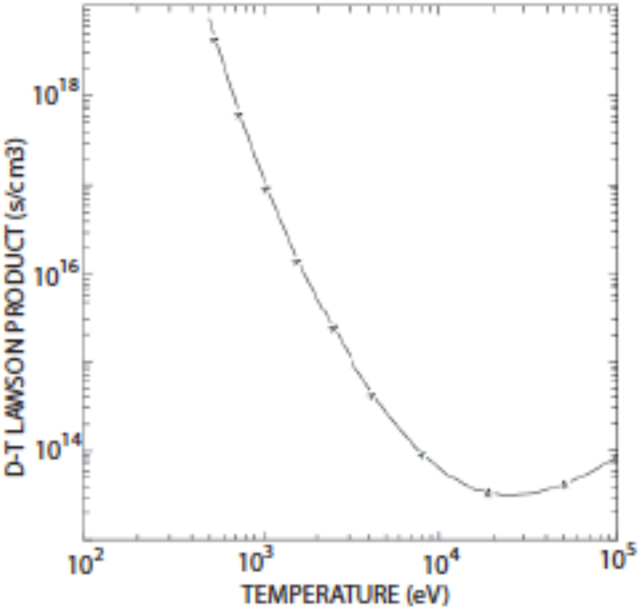
$$G = \frac{E_{FUS}}{3 n_o T_o} = G_{\max} n_o t \frac{\overline{\sigma v}}{2} \frac{1}{1 + \frac{\overline{\sigma v}}{2} n_o t}, \qquad G_{\max} = \frac{\varepsilon_{FUS}}{6 T_o}$$

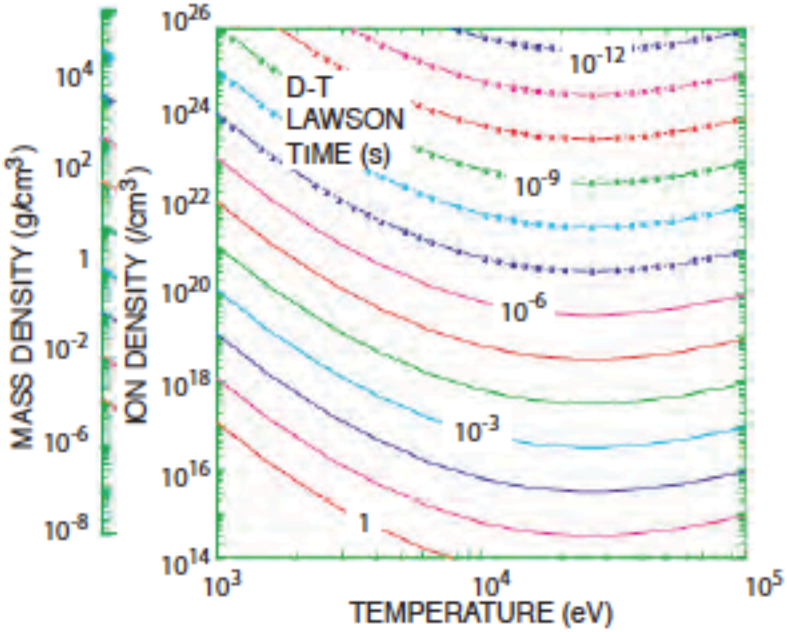
- **G depends only on $n_o t$, tells how long plasma must be "confined"**



“Scientific breakeven” (G=1) is a continuing goal of fusion research

- G = 1 when $n_o t = \frac{2}{\overline{\sigma v}} \frac{1}{G_{max} - 1} = L(T)$ = Lawson product
- Plasma must be confined for the “Lawson time” $t_L = L/n_o$ for G=1





- To burn 33% of fuel, plasma must be confined for $(1/2)(G_{max}-1)t_L$; to burn 50%, $(G_{max}-1)t_L$.