

# The fundamental parameter space of controlled thermonuclear fusion

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We apply a few simple first-principles equations to identify the parameter space in which controlled fusion might be possible. Fundamental physical parameters such as minimum size, energy, and power as well as cost are estimated. We explain why the fusion fuel density in inertial confinement fusion is more than  $10^{11}$  times larger than the fuel density in magnetic confinement fusion. We introduce magnetized target fusion as one possible way of accessing a density regime that is intermediate between the two extremes of inertial confinement fusion and magnetic confinement fusion and is potentially lower cost than either of these two. © 2009 American Association of Physics Teachers.

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## I. INTRODUCTION

The two commonly recognized paths to controlled thermonuclear fusion energy have proven to be long and costly. Physicists not working in the fusion field are generally aware of two approaches to achieving controlled fusion reactions, magnetic confinement fusion (MCF) and inertial confinement fusion (ICF), and that these approaches are embodied in two multi-billion dollar facilities known as ITER<sup>1</sup> (formerly an acronym for International Thermonuclear Experimental Reactor) for magnetic confinement fusion and NIF<sup>2</sup> (National Ignition Facility) for inertial confinement fusion. Most physicists are also aware that fusion reactions occur only at high temperatures, hence the name thermonuclear fusion. There is more limited awareness that the density of the fusion fuel (a deuterium-tritium mixture) in the two approaches differs by a factor of more than  $10^{11}$ . The fuel density in ITER will be about  $10^{14}$  ions/cm<sup>3</sup> (mass density  $\rho = 4.2 \times 10^{-10}$  g/cm<sup>3</sup>), and the hot spot of a NIF target will be greater than  $10^{25}$  ions/cm<sup>3</sup> ( $\rho = 42$  g/cm<sup>3</sup>). This very large density difference leads to different physics challenges, for example, contrast the challenge of steady state magnetic confinement of MCF with the hydrodynamic implosion stability of ICF. The problem-solving approach is also different; MCF relies heavily on empirically determined scaling laws, and ICF relies heavily on large-scale computer simulations.

Post has provided an excellent introduction to the history of fusion and an extensive list of references.<sup>3</sup> Post notes that “the references cited also will be seen to be sparse in the area of simple treatments, because most fusion publications aim at an audience of plasma specialists.” This paper is intended to alleviate this shortcoming. We will examine fusion research from a first principles perspective to identify the parameter space in which controlled fusion might be possible. By using a few simple equations, we will derive expressions for estimating physical parameters such as minimum size, energy, and power as well as cost. These expressions are used to create plots in the density-temperature plane that show the extreme differences between the conventional approaches.

Our intent is to provide a simple quantitative introduction to fusion research that is suitable for an introductory class on fusion. We also provide a unifying framework to make it possible for those familiar with one approach to be conver-

sant in any other approach. Our analysis explains why MCF must operate at low density and why ICF must operate at high density.

There is a general perception that MCF and ICF are the only two ways to approach fusion research. However, our analysis shows what is not commonly acknowledged, mainly, that the fusion parameter space is a continuum between and beyond the two extremes, MCF and ICF. The analysis leads naturally to ask “Is there anything in the density space between the two conventional approaches?” In answering this question, we will introduce a third, untried approach: magnetized target fusion.

There is also a general perception that fusion research must be expensive. Therefore, we also address the question “Are there any potentially lower cost approaches that have not yet been adequately explored?” We use the known costs of several fusion facilities to estimate the cost of facilities that would operate in the unexplored “fusion no-man’s land” between the two conventional extremes. Our analysis offers hope that fusion research does not have to involve very expensive facilities. Fusion might yet be possible at lower cost.

In the analysis that follows we introduce a number of symbols that are tabulated in Table I.

## II. BASIC FUSION PRINCIPLES

Thermonuclear fusion takes place in a plasma consisting of fusion-able ions and charge-neutralizing electrons. The fusion reaction rate between one type of ion at concentration  $n_1$  and a second type at concentration  $n_2$  is given by a Maxwellian-averaged cross-section,  $\overline{\sigma v}$

$$\frac{dn_1}{dt} = \frac{dn_2}{dt} = -\overline{\sigma v}n_1n_2, \quad (1)$$

where  $n_1$  and  $n_2$  represent the number of particles per cm<sup>3</sup>. The fusion energy production rate,  $Q_{FUS}$ , is determined by multiplying the reaction rate by  $\epsilon_{FUS}$ , the fusion energy released per reaction, that is,

$$Q_{FUS} = \overline{\sigma v}n_1n_2\epsilon_{FUS}. \quad (2)$$

Howerton<sup>4</sup> has listed 24 different fusion reactions, given the  $\epsilon_{FUS}$  for each reaction, and plotted and tabulated the  $\overline{\sigma v}$  for each reaction. Because the cross-section  $\overline{\sigma v}$  is a function of temperature only, the fusion energy production rate depends only on the temperature and ion density. Therefore, the two

Table I. Symbols used in text.

Parameter	Symbol	Units
Time	$t$	s
Plasma parameters		
Mass density	$\rho$	g/cm <sup>3</sup>
Ion species density	$n_1$	/cm <sup>3</sup>
Ion species density	$n_2$	/cm <sup>3</sup>
Deuteron density	$n_D$	/cm <sup>3</sup>
Triton density	$n_T$	/cm <sup>3</sup>
Total ion density	$n_i$	/cm <sup>3</sup>
Initial total ion density	$n_0$	/cm <sup>3</sup>
Temperature	$T$	J
Energy loss	$E_{loss}$	J/cm <sup>3</sup>
Energy loss rate	$Q_{loss}$	W/cm <sup>3</sup>
Thermal conductivity	$K$	(cm*s) <sup>-1</sup>
Thermal conduction loss rate	$Q_{TC}$	W/cm <sup>3</sup>
Radiation loss rate	$Q_{RAD}$	W/cm <sup>3</sup>
Ion mass	$m_i$	g
Electron mass	$m_e$	g
Pressure	$p_{atm}$	atm
Pressure	$p$	J/cm <sup>3</sup>
Ratio of plasma to magnetic pressure	$\beta$	none
Magnetic field	$B$	G
Thermal diffusivity	$X$	cm <sup>2</sup> /s
Fusion reaction quantities		
Fusion cross-section, generic	$\overline{\sigma v}$	cm <sup>3</sup> /s
Fusion energy per reaction, generic	$\varepsilon_{FUS}$	J
Fusion energy production rate, generic	$Q_{FUS}$	W/cm <sup>3</sup>
Fusion cross-section, D-T	$\overline{\sigma v}_{DT}$	cm <sup>3</sup> /s
Fusion energy per reaction, D-T	$\varepsilon_{DT}$	J
Fusion energy produced, D-T	$E_{DT}$	J/cm <sup>3</sup>
Fusion energy production rate, D-T	$Q_{DT}$	W/cm <sup>3</sup>
Lossless energy gain	$G$	none
Maximum lossless energy gain	$G_{max}$	none
Lawson time	$\tau_L$	s
Lawson product	$L$	s/cm <sup>3</sup>
Ratio of fusion rate to loss rate	$\phi$	none
Effective gain	$G_1$	none
Scientific breakeven time	$\tau_1$	s
Geometric Quantities		
Volume	$V$	cm <sup>3</sup>
Surface area	$S$	cm <sup>2</sup>
Characteristic dimension (size)	$a$	cm
Ratio $V/a^3$	$\varepsilon$	none
Ratio $V/(Sa)$	$\gamma$	none
Temperature gradient scale factor	$\alpha$	none
Initial radius	$r_0$	cm
Final radius	$r_f$	cm
System Parameters		
Minimum mass	$M$	g
Minimum energy	$E_{PLAS}$	J
Required heating rate	$P_{HEAT}$	W
Required heating intensity	$I_{HEAT}$	W/cm <sup>2</sup>
Required implosion velocity	$v_{IMP}$	cm/s
Capital cost	Cost	US\$
ITER plasma energy	$E_{ITER}$	J
NIF heating rate	$P_{NIF}$	W
Radial compression ratio	$C$	none

most important physical parameters of a fusion system are the temperature and the ion density.

A 50% deuterium (D), 50% tritium (T) mixture is generally preferred for first generation reactors because the D-T reaction has a higher cross-section than any other fusion reaction. The D-T cross-section reaches its maximum value at a temperature of approximately 60 keV. To reach a value of even 1% of the maximum, a temperature of 4 keV ( $5 \times 10^7$  K) must be reached and sustained. At such high temperatures the D-T mixture is a plasma.

If we assume for simplicity that the temperature does not change with time and that the fuel has no flow velocity, Eq. (1) can be integrated to give the total ion density ( $n_i = n_D + n_T = 2n_D = 2n_T$ , where  $n_D$  and  $n_T$  are the deuteron and triton density, respectively) as a function of time

$$n_i = \frac{n_0}{1 + \frac{\overline{\sigma v}_{DT}}{2} n_0 t}, \quad (3)$$

where  $n_0$  is the ion density at  $t=0$  and  $\overline{\sigma v}_{DT}$  is the cross-section for the D-T reaction. The corresponding fusion energy produced and the energy production rate are given by

$$E_{DT} = \varepsilon_{DT} n_0^2 \frac{\overline{\sigma v}_{DT}}{4} \frac{1}{1 + \frac{\overline{\sigma v}_{DT}}{2} n_0 t}, \quad (4)$$

$$Q_{DT} = \frac{dE_{DT}}{dt} = \varepsilon_{DT} n_i^2 \frac{\overline{\sigma v}_{DT}}{4}, \quad (5)$$

where  $\varepsilon_{DT} = 2 \times 10^{-12}$  J (17.6 MeV) is the energy produced in each D-T fusion reaction.

For fusion energy systems a key scientific and engineering figure of merit is the energy gain, that is, the ratio of the energy produced to some measure of the energy invested to produce the fusion energy. Although a number of definitions of gain can be found in the literature (see, for example, the references cited in Ref. 3), the definition appropriate for this paper is the ratio of the fusion energy produced to the initial plasma energy  $3n_0T$ , where  $T$  is the plasma temperature, that is,

$$G = \frac{E_{DT}}{3n_0T} = G_{max} n_0 t \frac{\overline{\sigma v}_{DT}}{2} \frac{1}{1 + \frac{\overline{\sigma v}_{DT}}{2} n_0 t}, \quad G_{max} = \frac{\varepsilon_{DT}}{6T}. \quad (6)$$

Although the cross-section increases with temperature (up until 60 keV), the maximum gain decreases; for example, at 4 keV the maximum gain is 733, but at 20 keV, the maximum gain is only 147.

From Eq. (6) the temperature dependence of the gain is through the cross-section and  $G_{max}$ , and the density dependence is only through the product,  $n_0 t$ . "Scientific breakeven" ( $G=1$ ) is a continuing goal of fusion research.  $G=1$  occurs when

$$n_0 \tau_L = \frac{2}{\overline{\sigma v}_{DT} G_{max} - 1} = L, \quad (7)$$

where  $\tau_L$  is the Lawson time and  $L$  is the Lawson product (as with gain, several closely related definitions of the Lawson product can be found in the literature; see, for example, the

references cited in Ref. 3). The Lawson product is typically given as  $10^{14}$  s/cm<sup>3</sup>, but as defined here, ranges from about  $5 \times 10^{14}$  s/cm<sup>3</sup> at  $T=4$  keV to a minimum of about  $3 \times 10^{13}$  s/cm<sup>3</sup> at  $T=25$  keV. A fusion plasma must be confined for the Lawson time, which varies inversely with density, to achieve breakeven.

Scientific breakeven is a necessary, but not sufficient, condition for energy applications of fusion. ITER and NIF are expected to be the first facilities to achieve scientific breakeven, ITER sometime after 2020 and NIF sometime after 2010. Because scientific breakeven has not yet been achieved, we will limit discussions in this paper to the conditions required to achieve it. We note that for reactors, the plasma must be confined for many Lawson times to produce net energy and allow additional fuel to burn, but we will not discuss conditions beyond scientific breakeven.

Plasma energy loss to cold surroundings is a major obstacle to fusion. When losses are taken into account, the effective gain is reduced, that is,

$$G_I = \frac{E_{DT} - E_{loss}}{3n_0T}, \quad (8)$$

where  $E_{loss}$  is the energy lost during the fusion process. If  $Q_{loss} = dE_{loss}/dt = \phi Q_{DT}$ , the effective gain is reduced by a factor  $1 - \phi$ . If we use Eqs. (5)–(8), we can express the confinement time required to reach scientific breakeven,  $G_I = 1$ , as

$$\begin{aligned} \tau_1 &= \frac{1}{n_0} \frac{2}{\bar{\sigma}v_{DT}} \frac{1}{(G_I)_{\max} - 1} \\ &= \tau_L \frac{G_{\max} - 1}{(G_I)_{\max} - 1} \\ &\approx \tau_L \frac{1}{1 - \phi} = N\tau_L. \end{aligned} \quad (9)$$

Equation (9) indicates that the time required to reach breakeven can be many times longer than the Lawson time given by Eq. (7). For example, if  $\phi=0.9$ , ten Lawson times are required. For the confinement time to be less than 1.5 Lawson times,  $\phi$  must be less than 0.33; that is, the fusion energy production rate must be at least three times higher than the loss rate. In addition to increasing the time required for breakeven, losses also mean that the plasma must be heated at a rate equal to  $Q_{loss}$  to sustain the plasma through its burn time.

### III. FUSION DENSITY-TEMPERATURE PARAMETER SPACE

By comparing loss and fusion rates we can identify the density-temperature combinations where fusion gain can be achieved, because  $Q_{DT} \geq Q_{loss}$ , that is,  $\phi \leq 1$ , is a minimum condition. In the simplest “classical” case the loss rate is given by

$$Q_{loss} = C_{RAD} n_i^2 T^{1/2} - \nabla \cdot (K \nabla T), \quad (10)$$

where the first term is Bremsstrahlung radiation (as given by Spitzer<sup>5</sup>) and the second term is thermal conduction (with  $K$  the thermal conductivity<sup>6</sup>). The radiation loss depends only on the density and temperature, whereas the  $K$  can also depend on the magnetic field strength (if it is present).

Because both the radiation and fusion rates depend on  $n_i^2$ , the ratio of the radiation term in Eq. (10) to the fusion rate in Eq. (5) is independent of the density. The fusion rate exceeds the radiation rate at a temperature greater than approximately 3 keV. Thus, achieving and sustaining a temperature of 3 keV is a minimum requirement for breakeven.

To estimate the last term of Eq. (10), the following approximations can be used:

$$\begin{aligned} Q_{TC} &\approx -\frac{1}{V} \int \nabla \cdot (K \nabla T) dV \\ &= -\frac{1}{V} \oint_S K \nabla T \cdot d\bar{S} \\ &\approx -\frac{S}{V} K \nabla T \approx \frac{KT}{\gamma \alpha a^2}, \end{aligned} \quad (11)$$

where  $V = \epsilon a^3$ ,  $V/S = \gamma a$ ,  $\nabla T \approx -T/\alpha a$ ,  $a$  is a characteristic dimension,  $V$  is the plasma volume,  $S$  is the plasma surface area,  $\epsilon$  and  $\gamma$  are geometric quantities (for example, for spheres,  $\epsilon = 4\pi/3$  and  $\gamma = 1/3$ ), and  $\alpha$  is a temperature-gradient scale factor that, as described later, depends on the form of  $K$ .

### IV. MINIMUM FUSION SYSTEM SIZE

Equations (10) and (11) can be used to determine the characteristic size (dimension) of a fusion system operating at a given value of  $\phi$

$$a^2 = \frac{KT}{\gamma \alpha \phi Q_{DT} - Q_{RAD}}, \quad (12)$$

where  $Q_{RAD}$  is the first term on the right hand side of Eq. (10). If  $\phi=1$ , Eq. (12) gives the minimum size at a specified density and temperature (and magnetic field if present), because  $\phi \leq 1$  is a necessary, but not sufficient, condition for net energy production. If  $\phi < 1$ , Eq. (12) gives the minimum size required to operate at or below the specified  $\phi$ . Systems larger than that given by Eq. (12) will operate at a smaller  $\phi$ , and smaller systems will operate at a larger  $\phi$ .

Once the minimum size required to operate at or below a specified value of  $\phi$  for a specified density-temperature (and magnetic field) is determined, important physical parameters such as the minimum fuel mass  $M$  and the minimum plasma thermal energy  $E_{PLAS}$  can be determined

$$M = n_i(m_i + m_e)\epsilon a^3, \quad E_{PLAS} = 3n_i T \epsilon a^3. \quad (13)$$

The minimum required heating rate to sustain the plasma, and the corresponding heating intensity, can also be determined

$$P_{HEAT} = (Q_{TC} + Q_{RAD})\epsilon a^3, \quad I_{HEAT} = \frac{P_{HEAT}}{S}. \quad (14)$$

Equations (12)–(14) allow us to estimate the fundamental parameters of a fusion system. These parameters depend on density, temperature, geometry ( $\alpha$ ,  $\epsilon$ , and  $\gamma$ ), the loss rate ratio  $\phi$ , and the thermal conductivity.

Under certain circumstances, the scaling relations given by Eqs. (12)–(14) can be simplified. If, in Eq. (12),  $\phi Q_{DT} \gg Q_{RAD}$ , we can ignore the radiation losses. By eliminating  $\bar{\sigma}v_{DT}$  using Eqs. (5) and (7) and noting that  $G_{\max} \gg 1$ , we can write the fusion energy production rate as  $Q_{DT} = 3T_0 n_i^2 / L$  and

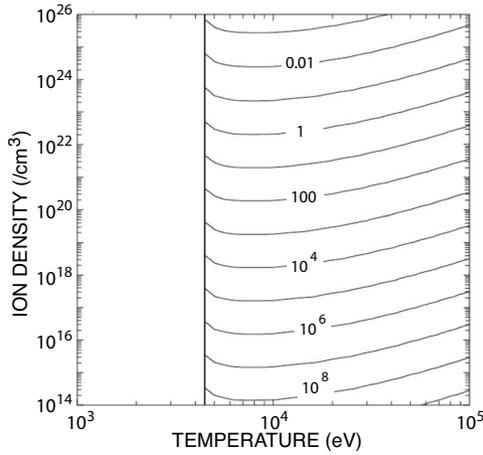


Fig. 1. The minimum system size (cm) for unmagnetized fuel (spherical geometry) operating at  $\phi \leq 0.2$ .

Eq. (12) becomes  $a^2 = XL(\alpha\gamma\phi n_i)$ , where  $X = K/(3n_i)$  is the thermal diffusivity. This basic relation, ignoring the factor of  $\alpha\gamma\phi$ , was first used by Siemon *et al.*<sup>7</sup> to demonstrate the strong density dependence that will be discussed in the following.

## V. PARAMETER SPACE FOR UNMAGNETIZED FUEL

For reasons that will become apparent, we will consider fuel density values ranging from  $10^{14}/\text{cm}^3$  to  $10^{26}/\text{cm}^3$ . Based on Eq. (9) and the subsequent discussion of confinement times, we will consider systems for which  $\phi \leq 0.2$ , so that the required confinement time is approximately the ideal Lawson time. As mentioned, the minimum size of a fusion system is given by  $\phi = 1$ , and hence, according to Eq. (12), a factor of  $\sqrt{5}$  smaller than the size for  $\phi = 0.2$ , but in this case an infinite number of Lawson times would be required.

Figure 1 gives the minimum size and Fig. 2 gives the minimum energy and power required for an unmagnetized system. To calculate the curves in Figs. 1 and 2 from Eqs. (12)–(14) we used spherical geometry ( $\epsilon = 4\pi/3$ ,  $\gamma = 1/3$ ).

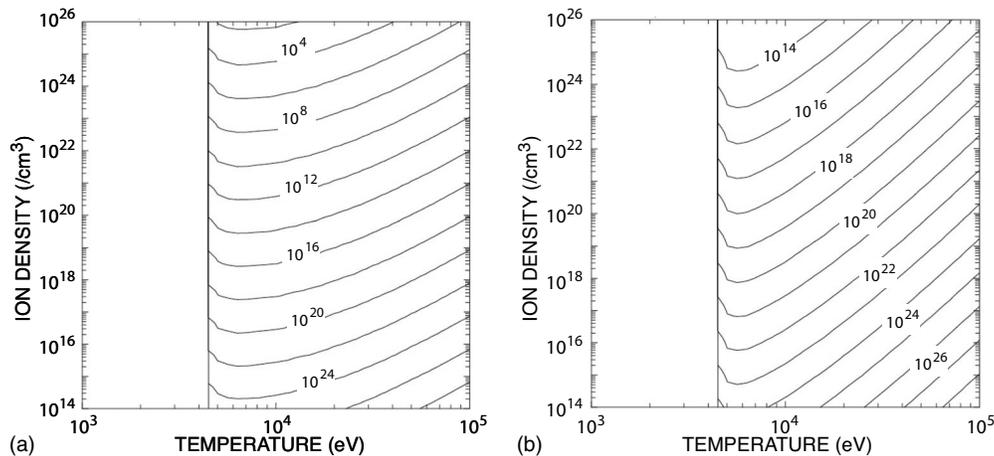


Fig. 2. (a) The minimum energy (J), and (b) the minimum power (W) for unmagnetized fuel (spherical geometry) operating at  $\phi \leq 0.2$ .

Following Eq. (27) in Ref. 2 we used  $\alpha = 1.75$ , which might be appropriate for the Braginskii  $T^{5/2}$  dependence of the thermal conductivity of an unmagnetized plasma.

Even at a density of  $10^{26}/\text{cm}^3$ , Fig. 2 shows that the power required for an unmagnetized plasma exceeds the U.S. total electricity generating capacity ( $\sim 1$  TW). As the density in Figs. 1 and 2 is lowered, the size and required energy and power increase rapidly. At a density below  $10^{16}/\text{cm}^3$ , the energy required becomes comparable to the energy received daily from the Sun ( $1.3 \times 10^{22}$  J). For a density at or below  $10^{14}/\text{cm}^3$ , the minimum size is comparable to Earth's diameter ( $10^9$  cm). Clearly, any approach using an unmagnetized plasma must operate at densities much higher than atmospheric density ( $2.4 \times 10^{19}/\text{cm}^3$ ).

The pressure in atmospheres is given approximately by

$$p_{\text{atm}} = 3.2 \times 10^{-15} n_i T_{\text{keV}}, \quad (15)$$

where  $T_{\text{keV}}$  is the temperature in keV. Hence, the pressures involved in Figs. 1 and 2 range from 0.32 atm in the lower right hand corner to  $3.2 \times 10^{13}$  atm in the upper right hand corner. We see that any terrestrial fusion system without a magnetic field will have to operate at extremely high pressures, that is, unmagnetized systems must be “pulsed;” steady-state operation would be impossible.

The electron thermal conductivity for an unmagnetized plasma exceeds the ion thermal conductivity by a few orders of magnitude.<sup>5,6</sup> Hence, it is the electron thermal conductivity that establishes the size, energy, and power minimums shown in Figs. 1 and 2. The dominant role of the electron thermal conductivity was recognized in the early days (1945) of fusion research. While at Los Alamos, Fermi<sup>8</sup> noted that

“A possible method for cutting down the conduction to the walls would be the application of a strong magnetic field, H ... so one would probably want to design the container elongated in the direction of H, or even toroidal ... with lines of force never leaving the deuterium ... rather large fields will be required ... thus a field in excess of 20,000 gauss would help reduce conduction loss.”

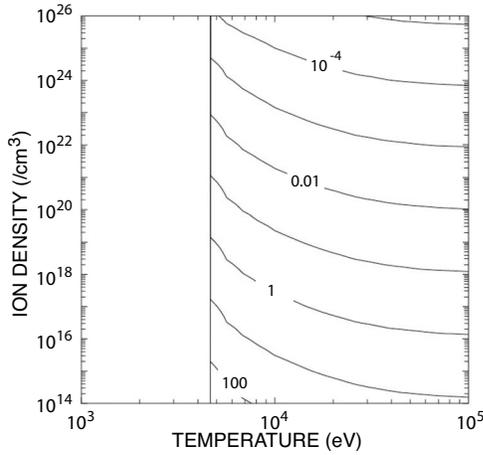


Fig. 3. The minimum system size (cm) for magnetized fuel (toroidal geometry,  $\beta=1$ ) operating at  $\phi \leq 0.2$ .

Fermi considered a field of 20 kG to be a “rather large” field. Fields even in excess of 1 MG can now be considered almost routine.<sup>9</sup>

## VI. PARAMETER SPACE FOR MAGNETIZED FUEL

If the system is magnetized in a configuration with closed magnetic field lines, the magnetic field  $B$  reduces the thermal conductivity and, hence, the energy loss rates from the system [Eq. (10)]. The thermal conductivity will be reduced approximately by a factor proportional to  $B^{-2}$ .<sup>5,6</sup> The reduction in energy loss will lead to corresponding reductions in the characteristic size, energy, and power of the required plasma. Figures 3 and 4 show examples of the reduction that can be obtained for the case  $\beta=1$ , where  $\beta$  is the ratio of the plasma pressure to the magnetic pressure,  $B^2/(8\pi)$ . To generate Figs. 3 and 4 we used geometric factors appropriate for a simple circular cross-section torus with a major-radius-to-minor-radius ratio of 3, that is,  $\epsilon=6\pi^2$  and  $\gamma=0.5$ . Just as we used Lindl’s analysis for the unmagnetized case, we have used a similar analysis to obtain  $\alpha=0.25$ , a value that is appropriate for the  $T^{-1/2}$  dependence of Braginskii’s cross-field thermal conduction for a strongly magnetized plasma.

For a given  $\beta$  the magnetic field required is

$$B = 2.8 \times 10^{-4} (n_i T_{keV} / \beta)^{1/2}, \quad (16)$$

where the unit of  $B$  is gauss. Hence, the magnetic field corresponding to each density-temperature point in Figs. 3 and 4 ranges from 2.8 kG in the lower left-hand corner to 28,000 MG in the upper right-hand corner.

Figures 3 and 4 show that the size and energy for magnetized plasma do not increase as rapidly as an unmagnetized plasma when the density is decreased and that, in contrast to an unmagnetized plasma, the power decreases as the density is decreased. In contrast to the unmagnetized case, the size, energy, and power requirements at the lower densities appear *a priori* to be feasible. For a density greater than  $10^{16}/\text{cm}^3$  both the magnetic and plasma pressures exceed 100 atm, so only the lower densities appear to be candidates for “steady state” operation.

## VII. CONVENTIONAL APPROACHES

In Secs. V and VI we identified the combinations of density, temperature, and magnetic field which, if maintained for a sufficiently long time, could lead to net energy gain, that is,  $G_I > 1$ . We showed that systems using unmagnetized fuel must operate at high density, whereas steady state systems must use magnetized fuel and operate at low density.

As mentioned, the two main approaches to fusion being pursued in the U.S. are MCF and ICF. These existing programs are attempting to access density extremes, which are determined in large part by the technologies chosen in each approach. We can use published parameters (such as density, temperature, and magnetic field) and the published characteristic dimension [or mass, related to the characteristic dimension through Eq. (13)] and our simple analysis to examine the characteristics of MCF and ICF and show the extreme differences between the two.

## VIII. INERTIAL CONFINEMENT FUSION

ICF attempts to compressionally heat an unmagnetized system to fusion temperatures by imploding a shell, that is, a pusher, around the fuel. The implosion is driven by a high-power “driver” such as a laser (for example, NIF) or particle beam. A “hot spot” in the center of an ICF target fuses and ignites a surrounding “cold fuel” layer in an effort to achieve high gain. We expect our simple analysis to apply to the NIF

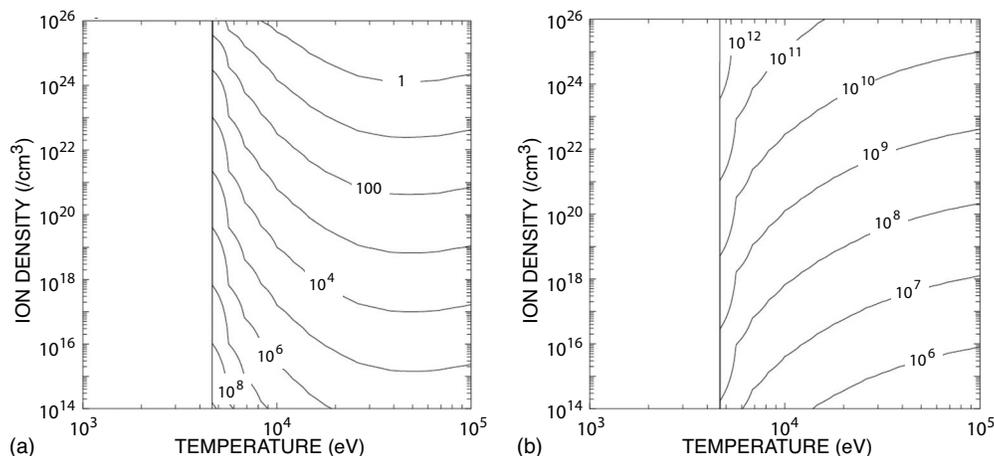


Fig. 4. (a) The minimum energy (J), and (b) the minimum power (W) for magnetized fuel (toroidal geometry,  $\beta=1$ ) operating at  $\phi \leq 0.2$ .

“hot spot” prior to the ignition time when self-heating by alpha particles (a product of the D-T fusion reaction) becomes dominant. To apply our simple expressions to NIF we note that the “hot spot” mass prior to ignition is ambiguous because of evaporation of the initially cryogenic cold fuel. From Ref. 2 we deduce that the hot spot reaches a temperature of approximately 8 keV, a density of approximately  $1.4 \times 10^{25}/\text{cm}^3$  ( $57 \text{ g}/\text{cm}^3$ ), and an areal density of  $0.2 \text{ g}/\text{cm}^2$  before self-heating becomes dominant. From these values we can derive that the hot spot has a characteristic dimension (radius) of  $3.5 \times 10^{-3} \text{ cm}$ . Substituting this value into Eqs. (13) and (14) results in a mass  $M$  of about  $10 \mu\text{g}$ , a thermal energy  $E_{\text{PLAS}}$  of 9 kJ, a heating requirement ( $P_{\text{HEAT}}, I_{\text{HEAT}}$ ) of  $1.1 \times 10^{14} \text{ W}$ ,  $7.5 \times 10^{17} \text{ W}/\text{cm}^2$ , and a volume of  $1.8 \times 10^{-13} \text{ m}^3$ .

As the implosion proceeds and ignition takes place, the density and the temperature of the hot spot will increase significantly, but even at the preignition density and temperature, the pressure of the fuel is approximately  $3.6 \times 10^{11} \text{ atm}$ . When ignition occurs, the pressure climbs even higher. Such pressure can only be contained by the inertia of the fuel and its surroundings, hence the name ICF. Because of the high pressures involved, ICF is a pulsed approach to fusion—energy is produced in a miniature nuclear explosion. Figure 1 shows that fusion is possible at lower density, and hence lower pressure. However, as shown in Fig. 2, when the density is lowered, the energy required quickly exceeds what can be coupled from a laser, even one as expensive as NIF. The latter is a 2 MJ laser powered by a 400 MJ capacitor bank, so only about 0.5% of the laser energy, and 0.0025% of the stored energy, is coupled to the hot spot.

The characteristic size of  $3.5 \times 10^{-3} \text{ cm}$  is about a factor of 2 larger than the minimum size shown in Fig. 1 at the same density and temperature. According to Eq. (11), this larger size reduces the thermal conduction losses. Thus, NIF operates at a loss ratio  $\phi$  of about 0.08. From Eq. (9) a burn time of only one Lawson time,  $6.6 \times 10^{-12} \text{ s}$ , is required to reach breakeven. However, the large inefficiencies of lasers in coupling to the fuel means that ignition of cold fuel must occur to produce energy comparable to the laser input, a necessary condition for energy applications. Hence, the hot spot fuel must burn for many Lawson times to produce sufficient energy to ignite the cold fuel. A discussion of fuel ignition, albeit of critical importance to ICF, is beyond the scope of this paper (see, for example, Refs. 2 and 10).

The fuel in ICF is heated by compression, that is, the imploding pusher does hydrodynamic work on the fusion fuel. The hydrodynamic heating rate per unit area is the fuel pressure multiplied by the implosion velocity. The hydrodynamic heating must be at least equal to the energy loss rate,  $Q_{\text{loss}}$ . Hence, the minimum implosion velocity required is given by

$$v_{\text{IMP}} = \frac{I_{\text{HEAT}}}{p} = \frac{I_{\text{HEAT}}}{2n_i T}. \quad (17)$$

For our NIF parameters Eq. (17) gives a minimum implosion velocity of  $21 \text{ cm}/\mu\text{s}$ . At this velocity the compressional heating only offsets losses and is not adequate to further increase the thermal energy of the fuel. Thus, in practice, a substantially higher velocity is required.

Shown in Fig. 5 is the minimum velocity required to offset losses for a hot spot mass of  $10 \mu\text{g}$ . Figure 5 indicates that the density-temperature space where compressional heating

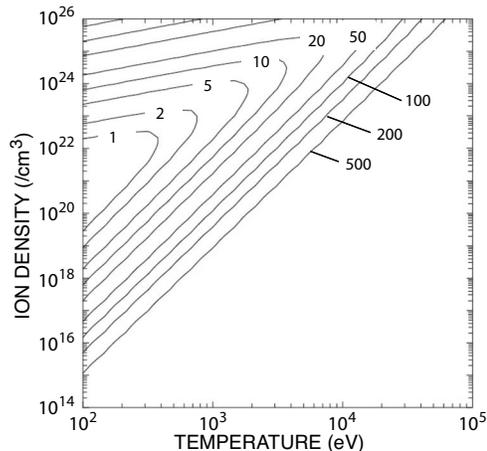


Fig. 5. The minimum velocity ( $\text{cm}/\mu\text{s}$ ) required to offset losses for the NIF mass,  $10 \mu\text{g}$ .

can be obtained is limited and careful control of the density-temperature trajectory is required. A velocity of  $1 \text{ cm}/\mu\text{s}$  can only reach a temperature of  $400 \text{ eV}$ , and a velocity greater than  $20 \text{ cm}/\mu\text{s}$  is required to reach a density of greater than  $10^{25}/\text{cm}^3$  at fusion temperatures. As discussed in Ref. 2 the NIF implosion velocity is about  $40 \text{ cm}/\mu\text{s}$ , and “the optimal velocity...is the primary determinant of the minimum size driver for ignition...; for both laser-driven and ion-driven indirect drive...the capsule implosion and burn physics are the same...”<sup>2</sup> This statement implies that any method for reducing the required velocity would reduce the driver size and, hence, the driver cost. We will return to this point later.

## IX. MAGNETIC CONFINEMENT FUSION

MCF is focused on tokamaks, a toroidal plasma configuration that uses a magnetic field with toroidal and poloidal components. The fuel pressure in tokamaks is only a few percent of the magnetic pressure. The goal of MCF is a steady-state fusion reactor, and when magnetic pressures and material strengths are considered, the pressure of the fusion fuel cannot be higher than a few atmospheres. In steady state, energy losses must be offset indefinitely by either external heating or self-heating (alpha particle deposition). In contrast to ICF, the magnetic field can enhance the deposition of alpha particles, potentially leading to ignition whereby external heating is not required. As with ICF, a discussion of fuel ignition is beyond the scope of this paper, as is a discussion of refueling in a steady-state reactor.

In principle, the only requirement for true steady-state operation is  $\phi < 1$ , because the initial energy investment used to define gain [Eqs. (6) and (8)] ultimately becomes negligible. A perfect magnetic confinement scheme would have no losses except radiation.

ITER will have a density of  $10^{14}/\text{cm}^3$ , an ion temperature of  $8 \text{ keV}$ , a toroidal field of  $50 \text{ kG}$ , a poloidal field of  $10 \text{ kG}$ , a volume of  $831 \text{ m}^3$ , and a surface area of  $683 \text{ m}^2$ .<sup>1</sup> The volume-to-surface ratio is the same as a simple circular-cross-section torus with aspect ratio of three, and a characteristic dimension (minor radius) of  $240 \text{ cm}$ , which is equivalent to ITER’s stated  $200 \text{ cm}$  minor radius because of its noncircular cross-section. There is some experimental evidence that only the poloidal field is effective in reducing losses, and in ITER the ratio of plasma pressure to poloidal

magnetic field pressure is about 0.65. Thus, Figs. 3 and 4, and the analysis leading to them, are applicable.

Using 240 cm and the published ITER parameters of  $10^{14}/\text{cm}^3$ , 8 keV, and 10 kG, Eqs. (10) and (11) give a 5 MW conduction loss and a 12 MW radiation loss. According to Ref. 1, the radiation loss rates are expected to be about 47 MW due to impurities and line radiation, and other losses are expected to be 87 MW, about 18 times greater than predicted by our simple model using only the poloidal field strength (or about 450 times larger than would be predicted using the full field strength of about 50 kG). Because the fusion production rate is 400 MW, ITER is expected to operate at a loss fraction  $\phi$  of 0.33. Fortunately, much of the losses will be made up by alpha particle heating, so the net external power that must be supplied is about 40 MW, allowing ITER to operate at a ratio of fusion power to external power of about 10.

It is beyond the scope of this paper to discuss the very complex problem of tokamak energy-loss processes and why only the poloidal field should be considered in determining loss rates. The actual anomalous energy losses (often called “transport”) is a subject of ongoing research. Fortunately, tokamaks have not exhibited Bohm losses,<sup>11</sup> where the thermal conductivity is reduced by a factor that is inversely proportional to the magnetic field strength, rather than inversely proportional to the square of the field strength as is the case for the “classical” thermal conductivity used in our analysis. The impact of Bohm thermal conductivity can be evaluated by substituting the Bohm value in Eqs. (10)–(14). If ITER losses were Bohm-like, Eq. (11) would predict that ITER’s conduction losses would be about  $10^{11}$  W, or 250 times the expected fusion power of 400 MW. Put another way, if ITER were to operate at  $\phi=0.2$  under Bohm conditions, its characteristic size [Eq. (12)] would be about  $10^4$  cm and the required energy [Eq. (13)] would be  $2.6 \times 10^{13}$  J, an amount comparable to the electrical energy produced in the U.S. in about 30 s.

## X. FUSION SYSTEM CAPITAL COST

The cost of fusion facilities appears to be determined primarily by the required plasma energy (ITER) or by the required plasma heating rate (NIF). Particularly in ICF, emerging technologies such as pulsed-power-driven z-pinch radiation sources, heavy-ion drivers, and solid-state lasers might lead to lower cost. At present, we postulate that the cost of fusion facilities can be estimated as follows:

$$\text{Cost} = c_1 E_{\text{PLAS}} + c_2 P_{\text{HEAT}} \approx \frac{\$10\text{B}}{E_{\text{ITER}}} E_{\text{PLAS}} + \frac{\$3\text{B}}{P_{\text{NIF}}} P_{\text{HEAT}}, \quad (18)$$

where  $E_{\text{ITER}}=320$  MJ and  $P_{\text{NIF}}=1.1 \times 10^{14}$  W. We will confirm the validity of Eq. (18) later, but first we examine its implications. Figure 6 gives the cost that would be predicted for a facility to access the parameter space of Figs. 3 and 4. Figure 6 offers a startling prediction: The cost of a fusion facility to access the region in between the extremes of MCF and ICF is potentially several orders of magnitude lower than ITER and NIF.

Figures 3, 4, and 6 imply that operation at  $5 \times 10^{20}/\text{cm}^3$ , 40 keV, and 40 MG ( $\beta=1$ ) requires a minimum size, mass, energy, and power of 54  $\mu\text{m}$ , 20 ng, 91 J, and 268 MW, respectively, at a cost of \$10K. Operation at such a small size,

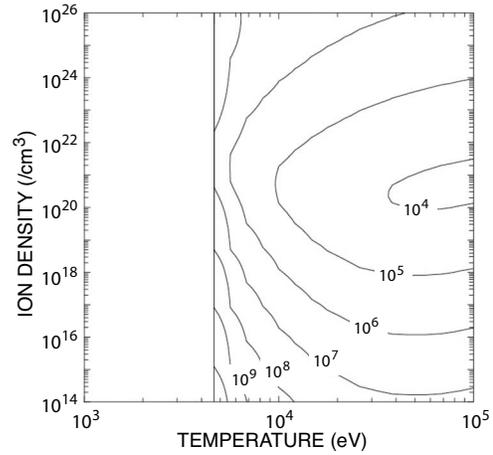


Fig. 6. The minimum facility cost (US \$) for magnetized fuel (classical thermal conductivity, toroidal geometry,  $\beta=1$ ) operating at  $\phi \leq 0.2$ .

high temperature, and high magnetic field could potentially involve technological challenges that are not reflected in the cost estimates. Therefore, let us consider something that could be accessed by existing technology (as we will discuss): 1.7 mg,  $10^{20}/\text{cm}^3$ , 8 keV, 1 MG ( $\beta=65$ ), and cylindrical geometry with a length-to-diameter ratio of 3 ( $\epsilon=6\pi$  and  $\gamma=0.75$ ). This mass is approximately the geometric mean between the NIF hot spot mass (10  $\mu\text{g}$ ) and the ITER mass (0.35 g) and the density is approximately the geometric mean between the ITER and NIF densities. Operation in this regime results in a loss ratio  $\phi=0.05$  and requires a radius, energy, and power of 0.6 cm, 1.6 MJ, and  $9.0 \times 10^{10}$  W, and most significantly according to Eq. (18), a cost of \$51M.

Figure 6 appears to dispel the common notion that fusion must be expensive. Therefore, we ask if Eq. (18) is valid. We note that the Tokamak Fusion Test Reactor (TFTR),<sup>12</sup> an ITER predecessor, had a plasma energy of 7 MJ. Hence, Eq. (18) would estimate the cost of TFTR as \$220M, which is comparable to its actual cost.

To check the validity of the \$51M estimate for the intermediate 1.7 mg,  $10^{20}/\text{cm}^3$ , 8 keV, 1 MG cylindrical case, we consider the possibility of reaching fusion temperatures by compressing the fuel hydrodynamically with a cylindrical pusher, commonly called a “liner.” Equation (17) implies that a minimum velocity of 0.038 cm/ $\mu\text{s}$  is required, which is less than 1% of the NIF velocity. (As implied in Ref. 2, this reduced velocity reduces the driver requirements and, hence, should reduce the driver cost.) If the compression is approximately adiabatic, the density, temperature, and magnetic field would increase as  $C^2$ ,  $C^{4/3}$ , and  $C$ , where  $C = r_0/r_f$  is the radial compression ratio, that is, the ratio of the initial radius  $r_0$  to the final radius  $r_f$ . If we limit  $C$  to 10 (note that the NIF radial compression, or convergence, ratio is a difficult-to-achieve 30–40), then the initial radius of the liner must be 6 cm. Because we used a length-to-final diameter ratio of 3, the length of the liner must be 3.6 cm. If we assume a 10%–20% coupling of liner energy to the plasma within the liner, the liner would have to have a kinetic energy of 8–16 MJ with a velocity of, say, 0.1 cm/ $\mu\text{s}$  or more.

Liners with dimensions and energy similar to those required to access this intermediate parameter space are the type for which the Atlas<sup>13</sup> capacitor bank was designed, although not in a fusion context. The cost of Atlas was ap-

proximately \$50M, so the estimate of \$51M from Eq. (18) is confirmed as the approximate cost of a facility to access the intermediate density range. Therefore, we conclude that Eq. (18) and Fig. 6 give a reasonable estimate of the cost of facilities in the entire range of densities and temperatures.

## XI. MAGNETIZED TARGET FUSION

If we consider the intermediate 1.7 mg,  $10^{20}/\text{cm}^3$ , 8 keV, 1 MG case as resulting from a quasi-adiabatic compression of a magnetized plasma by a liner, and limit the radial compression ratio to 10, a plasma with an initial density, temperature, and magnetic field of  $10^{18}/\text{cm}^3$ , 371 eV, and 100 kG, respectively, would be required. These parameters are close to the parameters that have been obtained in the Russian MAGO program.<sup>14</sup> We conclude that facilities to access intermediate densities are not only much less expensive than NIF and ITER, but such facilities might already exist, that is, the \$50M Atlas might serendipitously be an appropriate machine for accessing the intermediate density regime if the liners already demonstrated on Atlas were coupled with a MAGO-like plasma formation system.

In 1991, one of the authors (IL) coined the name “magnetized target fusion” (MTF) to describe approaches that use a liner/pusher implosion system to compress an intermediate density magnetized plasma to fusion conditions. As currently envisioned, magnetized target fusion is a two-step process: (1) Formation of a magnetized plasma either within an implodable container or external to the container, in which case the plasma must be injected into the container after formation and (2) implosion of the shell surrounding the preformed plasma. Several combinations of plasma formation methods and implosion drivers appear possible.<sup>15</sup> Magnetized target fusion is a subset of the more broader class of “magneto-inertial fusion” concepts, which would include such concepts as the non-imploded, magnetically insulated inertial confinement approach of Kammash.<sup>16</sup>

Although we will mention a few magnetized target fusion approaches, it is beyond the scope of this paper to review its history. A number of examples can be found in the last five decades where it was recognized that the intermediate density regime was attractive. None of these approaches received enough funding to reach technical maturity, and many efforts were stopped as funding was diverted to the more conventional MCF and ICF approaches.

Lost in history is that the first fusion neutrons produced in the U.S. particle beam fusion program came from a magnetized target imploded by an electron beam machine at Sandia National Laboratory.<sup>17</sup> Although these “ $\Phi$ -targets” were promising, and in spite of calculations<sup>18</sup> that showed that high-gain was possible with magnetized targets requiring significantly reduced driver requirements and hence reduced costs, Sandia abandoned magnetized targets, evidently believing that conventional, unmagnetized targets offered a more optimum path. The U.S. eventually abandoned electron and light ions as implosion drivers, because they could not meet the requirements for unmagnetized targets (but might have been able to meet the requirements for magnetized targets). Work on heavy-ion drivers continues, and magnetized targets could potentially reduce the driver requirements needed.

In 1983, Lindemuth and Kirkpatrick formulated a simple implosion model that made it possible to survey the parameter space in which magnetized targets might work.<sup>19</sup> A

simple approximation to thermal conduction losses analogous to Eq. (11) was used, and two-dimensional calculations confirmed the use of  $\alpha=0.2$  in the simpler model. The simple model continues to serve as a guide for more detailed, multi-dimensional magnetohydrodynamic computations. At the time the model was formulated, lasers were considered the most likely drivers, and plasma creation was considered a challenge. Therefore, the model was applied to low energy (for example, 10 kJ) implosions and low initial plasma temperatures (for example, 50 eV), values that can be significantly exceeded with present-day drivers and plasma formation systems.

When the Cold War ended, it became possible for Russian scientists to share their accomplishments with the global scientific community, and we learned of the MAGO approach to what we now call magnetized target fusion.<sup>20</sup> Scientists from the All-Russian Scientific Research Institute of Experimental Physics at Sarov (formerly Arzamas-16), building on ideas originated by Andre Sakharov, are attempting to access the intermediate density by using MAGO magnetized plasma formation methods and by using relatively low-cost disk explosive magnetic flux compression generators<sup>21</sup> as implosion drivers. When single-shot flux compression generators are considered, it is possible that Eq. (18) has overestimated the cost of accessing an intermediate density.

Without going into detail, we emphasize that existing technology might allow access to an intermediate density space that covers several orders of magnitude in density. Magnetized target fusion is not merely the addition of a magnetic field to a conventional ICF target. In addition to the all-important aspect of cost, this intermediate density region appears to have many attractive attributes when compared with ICF. These attributes include larger and more easily fabricated targets, reduced radial compression, no driver pulse shaping, no carefully timed shocks, longer burn times, and magnetic-field enhanced alpha particle deposition. Because the intermediate space may be accessed using very highly efficient pulsed power drivers such as Atlas, magnetized target fusion reactors should not require high-gain as is essential for conventional ICF targets, although high-gain magnetized targets ignited by a magnetized hot-spot appear possible.<sup>22</sup> No magnetized target fusion reactor studies have been performed, but the work of Olson<sup>23</sup> on evaluating a z-pinch driver for conventional, x-radiation driven targets is very synergistic with magnetized targets.

As we have discussed, energy losses from tokamak plasmas are more than an order of magnitude higher than predicted based on the classical thermal conductivity used in our analysis, so the possibility of increased losses in the intermediate density range cannot be discounted. However, experiments<sup>24</sup> and numerical computations<sup>25</sup> suggest that the losses should be classical. Regardless, conventional wisdom suggests that the losses are unlikely to be larger than Bohm losses.

Whereas Bohm thermal conductivity is fatal at ITER densities, for the  $10^{20}/\text{cm}^3$ , 8 keV magnetized target fusion case, Bohm thermal conductivity increases the loss ratio  $\phi$  from 0.05 to 0.37, and the minimum velocity increases to 0.28 cm/ $\mu\text{s}$ . To operate with  $\phi=0.2$ , the energy and cost would go up by about a factor of 3 (4.6 MJ, \$168M) unless the magnetic field is increased. Figure 7 shows cost estimates for a magnetic field of 5 MG and Bohm losses. Even with Bohm losses, the lowest cost region of density-temperature space is at a density that is approximately the geometric

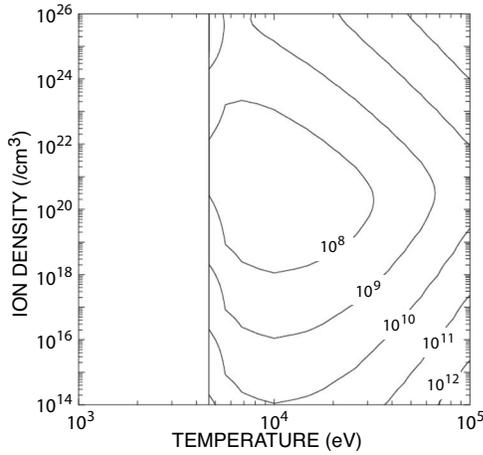


Fig. 7. The minimum facility cost (US \$) for magnetized fuel under Bohm conditions (cylindrical geometry,  $B=5$  MG) operating at  $\phi \leq 0.2$ .

mean between MCF and ICF. The space in which the cost is less than \$100M covers a factor of about  $10^4$  in density, and the space in which the cost is less than \$1B is even more expansive. In this intermediate parameter space the required implosion velocities are significantly less than required in ICF.

In spite of its apparent low cost, the intermediate space has yet to be adequately explored. However, the Los Alamos National Laboratory and the Air Force Research Laboratory, with support from the University of Nevada at Reno, are planning an experiment that will couple a field-reversed-configuration plasma with a liner driven by the Shiva Star capacitor bank. Suitable field-reversed-configuration target plasma formation and adequate liner symmetry and convergence have been demonstrated experimentally.<sup>26</sup> We anticipate that work being conducted under the OFES/NNSA High Energy Density Laboratory Physics initiative<sup>27</sup> will address many issues that are relevant to magnetized target fusion, for example, the behavior of materials under high magnetic field conditions. We note that Jones *et al.*<sup>28</sup> concluded that magnetized target fusion research is not a nuclear proliferation concern, although they doubt the energy potential of magnetized target fusion (and NIF).

## XII. CONCLUDING REMARKS

Our analysis has provided a framework for comparing various approaches to fusion. The simple analysis we have given offers a general understanding of the extreme differences between the two conventional approaches to controlled fusion, magnetic confinement fusion and inertial confinement fusion. We showed that magnetic confinement and inertial confinement fusion have a limited parameter space in which to operate: Steady-state operation forces magnetic confinement fusion to operate at the low end of the density spectrum, and the constraint of unmagnetized fuel forces inertial confinement fusion to operate at the high end.

We have raised the possibility that fusion might be possible at low cost by accessing the broad intermediate density range using an approach known as magnetized target fusion. Compared to the conventional MCF and ICF approaches, magnetized target fusion is a qualitatively different in terms of densities, pressures, time and length scales, and technologies. Any of these approaches may encounter significant

Table II. Fundamental physical parameters and cost for fusion systems discussed in text.

	ITER	MTF example	NIF
Geometry	Toroidal	Cylindrical	Spherical
Cost (\$M)	10,000	51	3000
$n_i$ (/cm <sup>3</sup> )	$10^{14}$	$10^{20}$	$1.4 \times 10^{25}$
$\rho$ (g/cm <sup>3</sup> )	$4.2 \times 10^{-10}$	$4.2 \times 10^{-4}$	57
$T$ (keV)	8	8	8
$p$ (atm)	2.6	$2.6 \times 10^6$	$3.6 \times 10^{11}$
$B$ (kG)	50	1000	0
$\tau_L$ (s)	0.9	$9 \times 10^{-7}$	$6.6 \times 10^{-12}$
$M$ (mg)	350	1.7	0.01
$a$ (cm)	240	0.6	$3.5 \times 10^{-3}$
$V$ (m <sup>3</sup> )	$8.3 \times 10^2$	$4.0 \times 10^{-6}$	$1.8 \times 10^{-13}$
$E_{plas}$ (J)	$3.2 \times 10^8$	$1.6 \times 10^6$	$9.3 \times 10^3$
$P_{heat}$ (W)	$1.3 \times 10^8$	$9.0 \times 10^{10}$	$1.1 \times 10^{14}$
$I_{heat}$ (W/cm <sup>2</sup> )	18	$1.0 \times 10^{10}$	$7.5 \times 10^{17}$

roadblocks on the way to practical fusion energy, but the approaches are so different that the roadblocks applicable to any one approach are not likely to apply to the others. Of the three approaches, magnetized target fusion is the least developed. In spite of some physics uncertainties, no insurmountable obstacles have been identified, and the technology required for achieving scientific breakeven and beyond appears to exist. Given the importance of energy to the future of society, a serious examination of all approaches is warranted.

Table II compares the magnetized target fusion example discussed in this paper with NIF and ITER and shows the extreme differences between MCF and ICF, differences that are exemplified by the factor of  $10^{15}$  difference in volume and the factor of  $10^{16}$  in intensity.

The intermediate density regime may be accessible by approaches other than magnetized target fusion. We invite readers interested in fusion to extend our analysis to other regions in the vast, unexplored fusion parameter space, and to identify new ways to access that parameter space. We will make our simple computer code available to enable exploration of the sensitivity of our results to the various parameters. Those interested in access to this code should contact us.

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One of the authors (IL) would like to acknowledge that he first became aware of fusion possibilities through discussions with Richard F. Post and that he first became aware of the possibility of an intermediate parameter space through discussions with Joseph S. Pettibone and the late Paul C. Wheeler. The authors' present understanding of the potential of magnetized target fusion as the lowest cost pathway is a result of their many interactions with our Russian and American colleagues. The Russian DEMG and MAGO systems, due to the genius of the late Vladimir K. Chernyshev<sup>29</sup> and his colleagues, have stimulated the authors' thinking and the thinking of many scientists throughout the world.

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