

# Tri-Alpha structures in $^{12}\text{C}$

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HIγS PROGRAM

# Development of an aneutronic Fusion Reactor Based upon the $^{11}\text{B} (p,\alpha)\alpha\alpha$ reaction

***Tri-Alpha Energy, Inc. –Foothills Ranch, Ca.***

*Controlled fusion in a field reversed  
configuration and direct energy conversion.*

(US patent: 7459654)

**The  $^{11}\text{B} (p,\alpha)\alpha\alpha$  reaction at low energies is dominated by the 2- T=1 resonance at 675 keV which has a width of 300 keV ( $\Gamma_p = 150$  keV,  $\Gamma_{\alpha 1} = 150$  keV).**

**The Q-value of the reaction is 8.58 MeV ( $E_{\text{in (cm)}} \times 14$ ).**

# The $^{11}\text{B} (p,\alpha)\alpha\alpha$ reaction

When 675 keV protons strike  $^{11}\text{B}$ , a resonance in  $^{12}\text{C}$  is formed at 16.6 MeV having a width of 300 keV.

This resonance decays by emitting a *primary*  $\alpha$ -particle, leading to the first excited state of  $^8\text{Be}$  which is 3 MeV above the ground state. This state decays into two *secondary*  $\alpha$ -particles.  $\sigma(\alpha_1) = 600 \text{ mb!}$

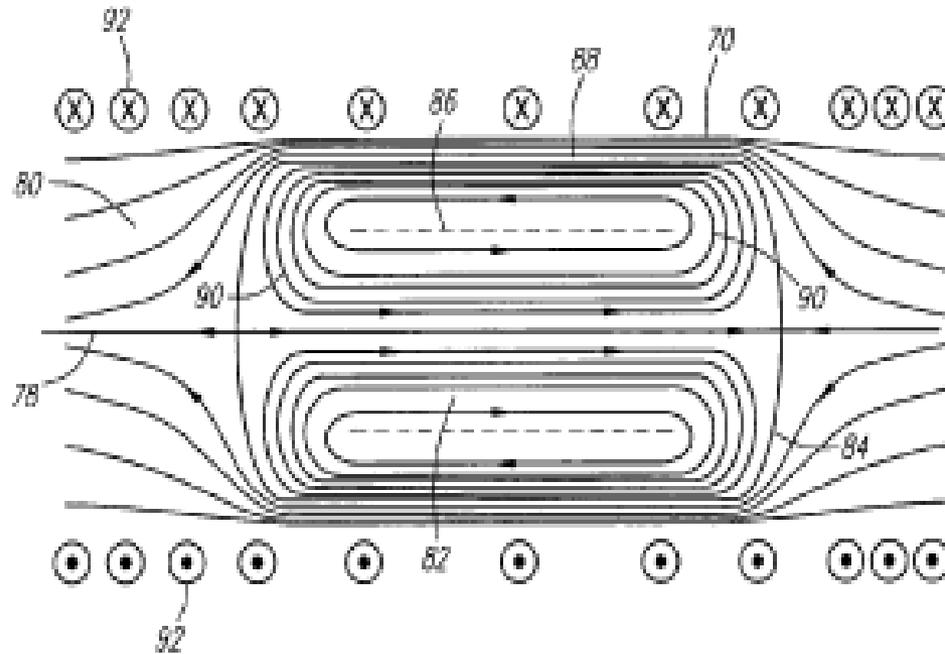
The state-of-the-art understanding of this reaction came from a 1987 study which “proved” that the reaction yielded one high energy *primary*  $\alpha$ -particle, and two low energy *secondary* ones.

# Tri-Alpha Energy Corporation

We have proven that this is incorrect!

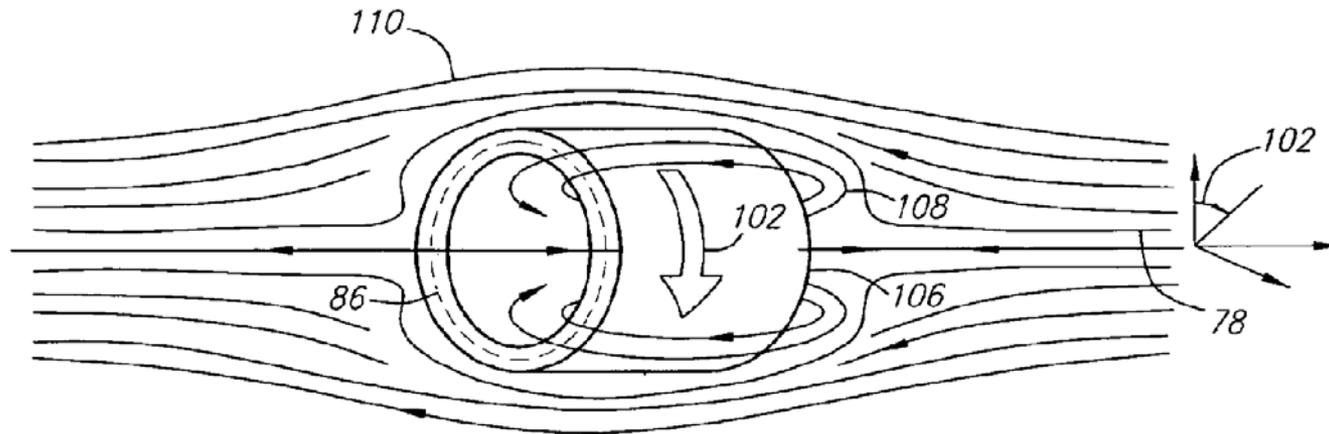
But first—a few words on the concepts which underlie the aneutronic fusion reactor presently being developed at Tri-Alpha, Inc.

# Magnetic Field map of the Field Reversed Configuration



*FIG. 2*

Annular layer of plasma rotates around the null surface (86).  
Collisions between injected p's and  $^{11}\text{B}$  create "high energy"  
alpha particles. Relative energy of protons to  $^{11}\text{B}$  is about 600  
keV.



The direct energy conversion system utilizes an Inverse Cyclotron Converter (ICC). D's are replaced by four semi-cylindrical electrodes. Energy is removed from the alphas as they spiral past the electrodes connected to a resonant circuit.

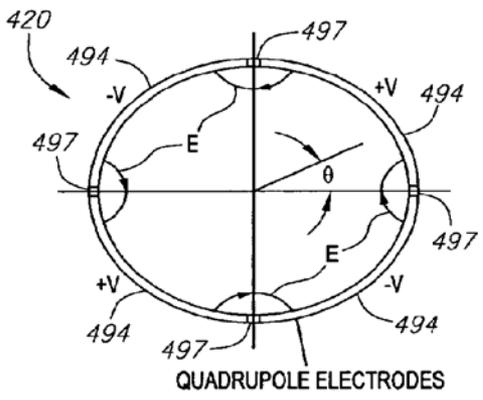


FIG. 19B

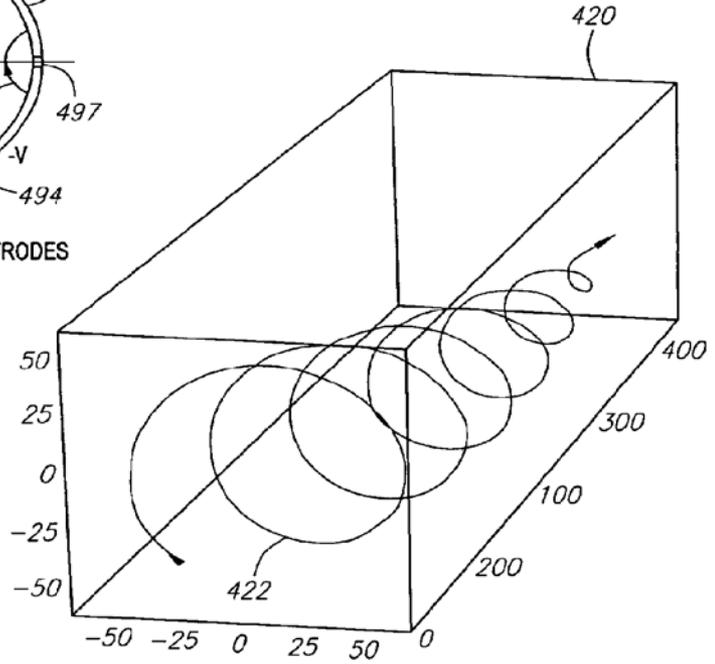


FIG. 19C

***The design of a 100 MW reactor is underway.  
Test “shots” to demonstrate plasma  
confinement are in progress.***

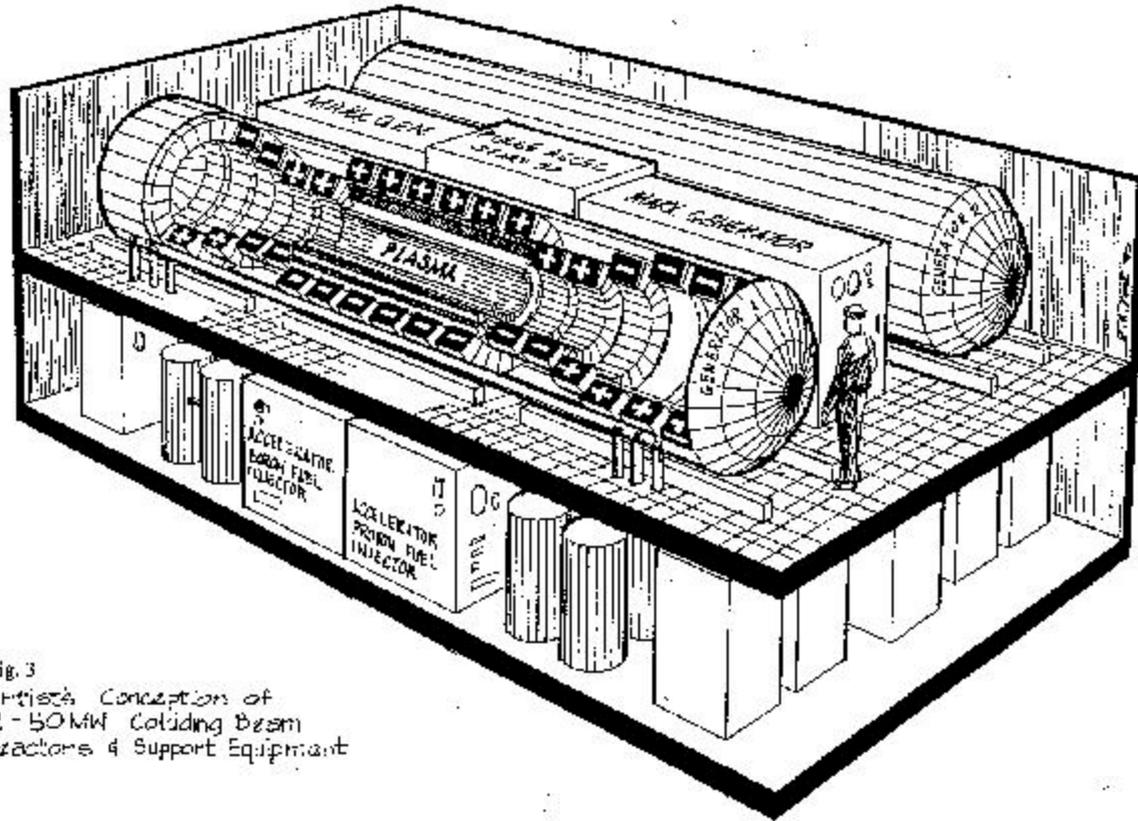


Fig. 3  
Artist's Conception of  
2-50 MW Colliding Beam  
Reactor & Support Equipment

# *Brief History of the $^{11}\text{B} (p, \alpha)\alpha\alpha$ reaction*

The history of this reaction is almost as long as the history of nuclear physics itself.

**Lord Rutherford** studied the reaction over 75 years ago at ~200 keV at the Cavendish Laboratory, measuring the ranges of particles coming from the target. In his *1933 Proceedings of the Royal Society* paper he states:

***“we might anticipate that the most probable mode of disintegration would be for the three  $\alpha$ -particles to escape symmetrically with equal energies”.***

***(Proc. Roy. Soc. Lond. A 141, 259 (1933).)***

Three years later (1936), Dee and Gilbert, also of the Cavendish, published the results of their expansion chamber studies of the  $^{11}\text{B} (p, \alpha)\alpha\alpha$  reaction. Their 300 keV studies led them to conclude:

***“the common mode of disintegration is into two [alpha] particles which proceed at angles of  $150^\circ$  to  $180^\circ$  relatively to one another, the third particle receiving very little energy”***

***The track images shown in their paper strongly support this interpretation.***

***(Proc. Royal Soc. Lond. A 154, 279 (1936).)***

This photo shows the typical mode of disintegration of boron into three  $\alpha$ -particles. A and B are emitted in nearly opposite directions while the third  $\alpha$ -particle C receives very little energy and barely emerges beyond the beam of scattered protons.

Downloaded from [rspa.royalsocietypublishing.org](http://rspa.royalsocietypublishing.org) on October 3, 2012

*Dee and Gilbert*

*Proc. Roy. Soc. A, vol. 154, Plate 5*

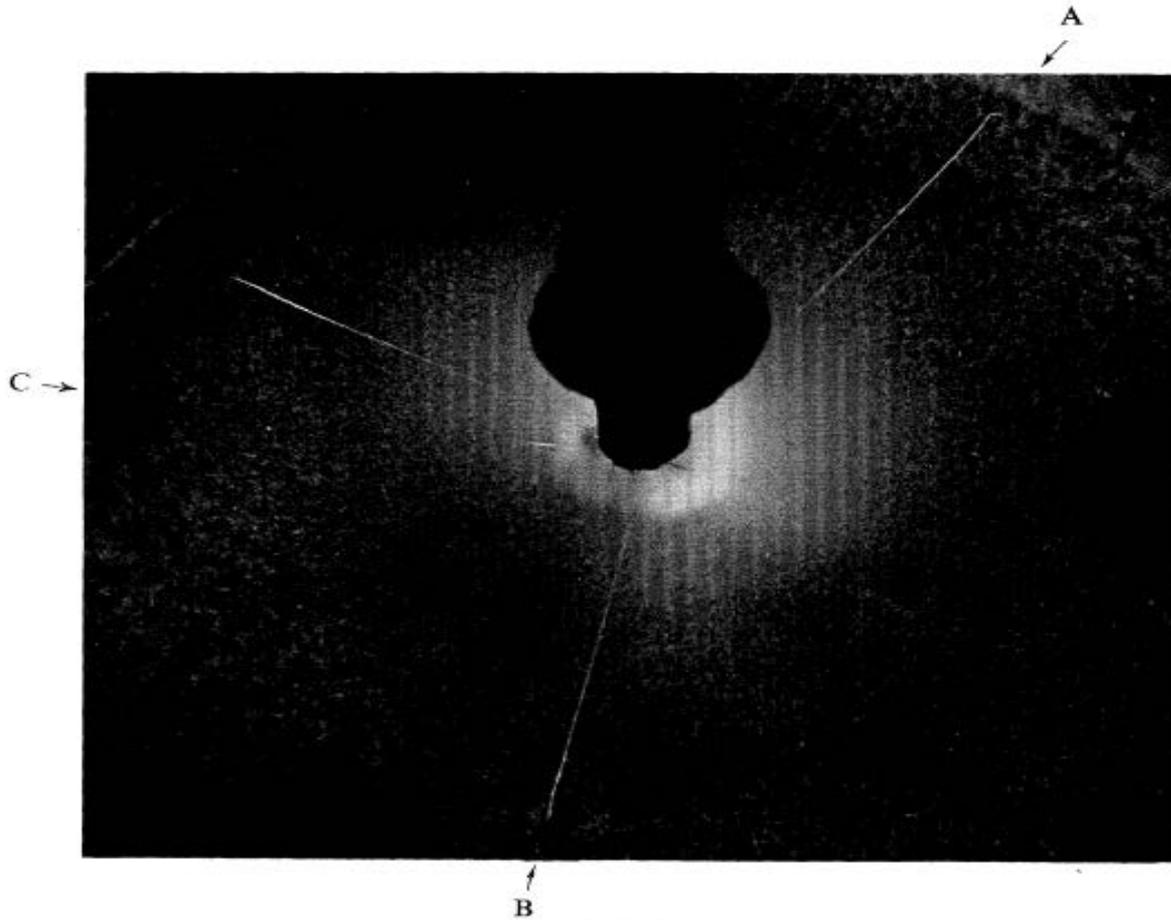


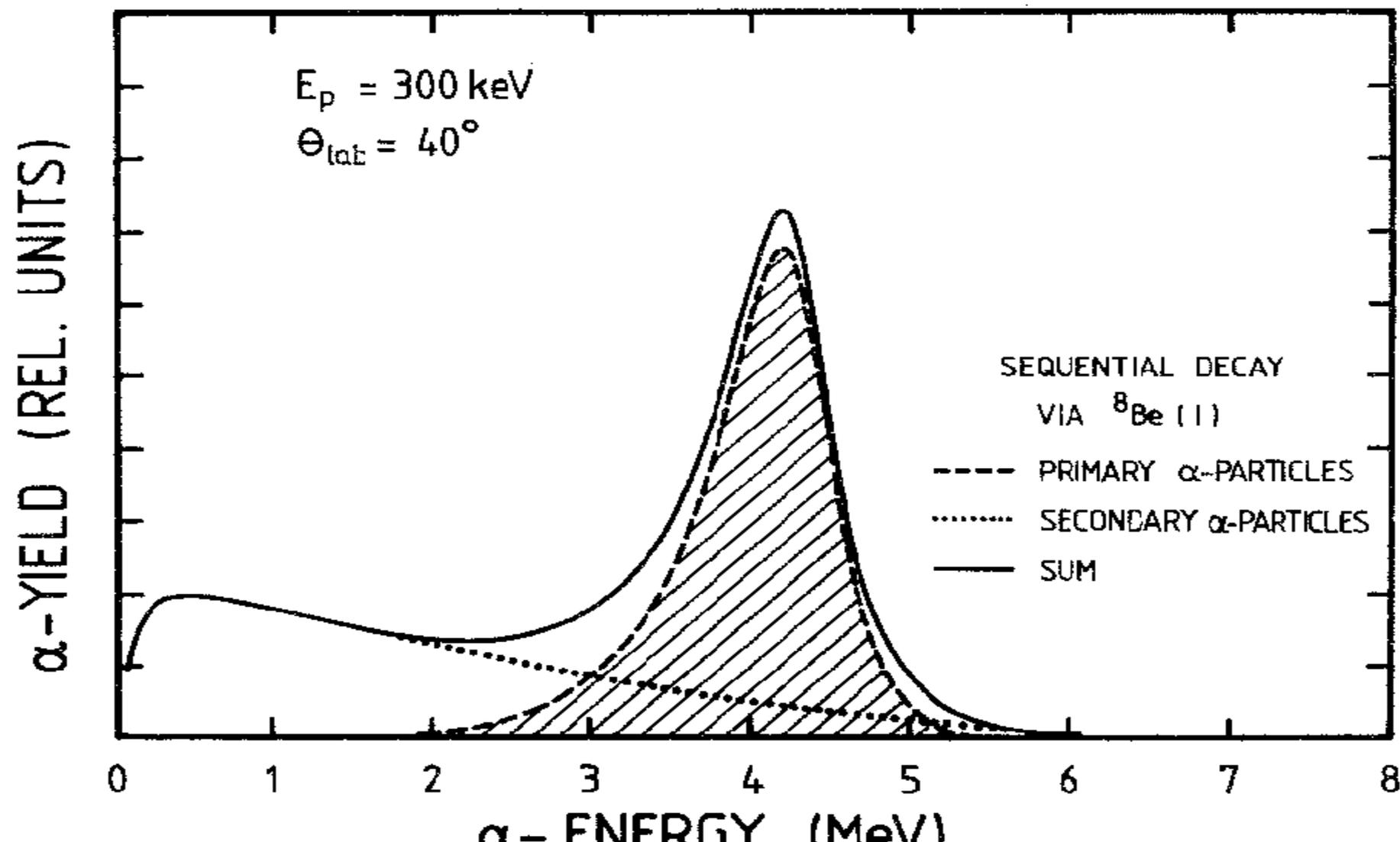
FIG. 6.

## *Brief History*

**Fast forward to 1987.** Of course many experimental and theoretical works were performed in the interim. But the work of *Becker, Rolfs and Trautvetter (Z. Phys. A327, 341 (1987))* established the *modern view* of this reaction:

***“The reaction mechanisms have been studied via kinematically complete coincidence measurements showing that the reaction proceeds predominantly by a sequential decay via  $^8\text{Be}$ ”***

**Primary and secondary distributions from Becker et al. The two are about equal, leading to the factor of "2" in determining the cross section.**



# ***The $^{11}\text{B}(p,\alpha)\alpha\alpha$ reaction***

***“We want to know the energy and location of every outgoing alpha particle”***      *Tri-Alpha Energy Inc.*

# Results of measuring the $\alpha$ -yields as a function of incident proton and outgoing $\alpha$ -energies.

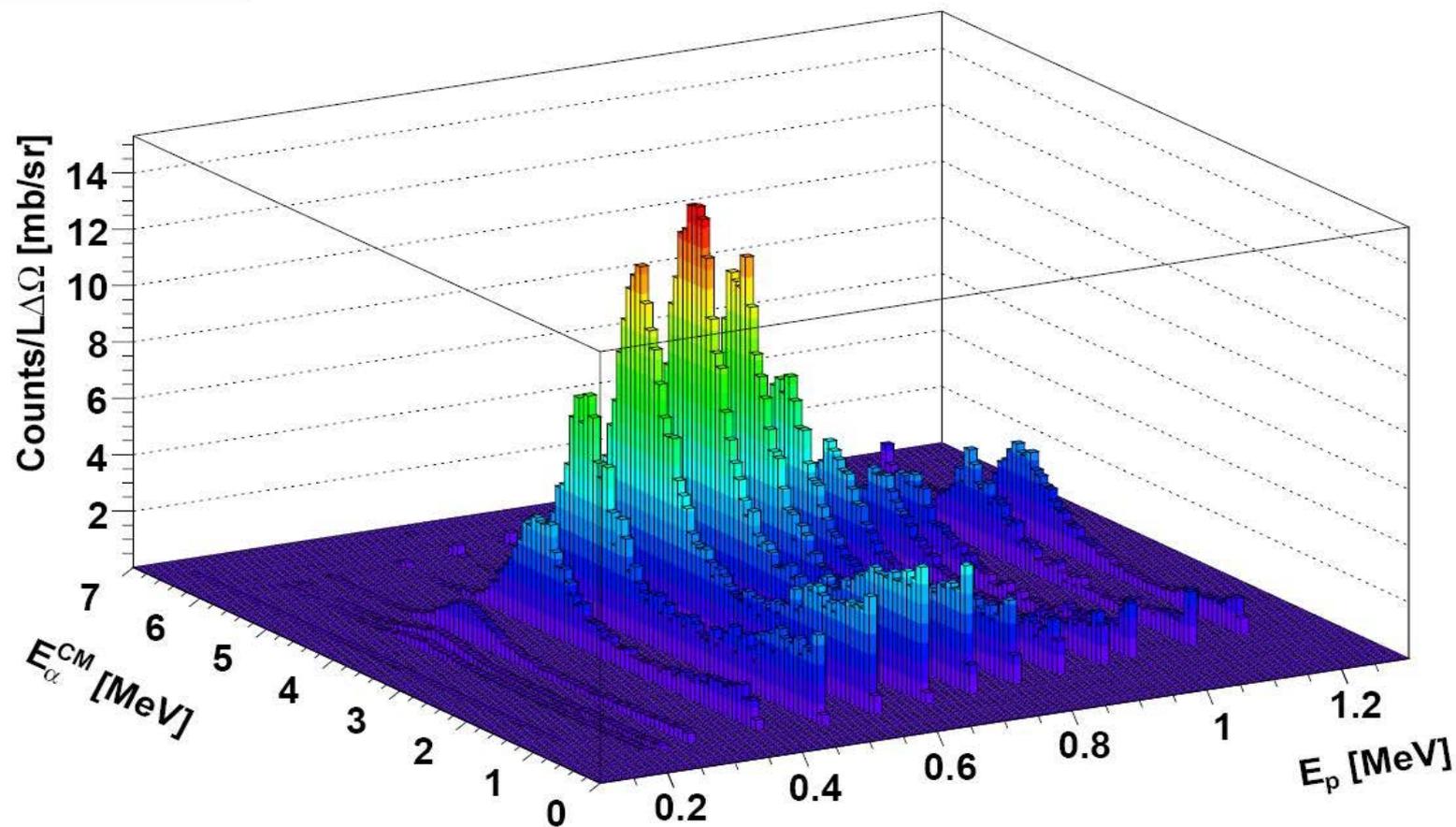
Previous results of Becker et al. indicated that only **two** alphas were observed in finite detectors. **We find 3.** NACRE assumed 2 in its Astrophysical S-factor compilation for this reaction. (***This needs correcting***)

*As will be shown later, the number of alphas is a function of both the proton energy and the alpha energy.*

To avoid this issue, we plot Counts instead of cross section, normalized to the integrated luminosity and solid angle. Cross sections can be obtained by dividing by the expected number of alpha particles, but **the absolute number of  $\alpha$ 's is what is important to Tri-Alpha.**

# Results showing the dominant 0.675 MeV resonance

$E_{\alpha}^{CM}$  vs.  $E_p$



# Two-step Reaction Simulation

Use **experimental**  $\alpha$ - $\alpha$  phase shifts to generate the resonance line shape of  ${}^8\text{Be}$  (2+). A pure single level assumption allows correction for potential scattering using hard-sphere phase shifts.

$$\delta_2 = -\phi_2 + \tan^{-1} \left( \frac{\Gamma/2}{E_R + \Delta_R - E} \right)$$

The second term is what we are after.

$\phi_2$  is the “hard-sphere” phase shift...easily calculated.

This avoids having to compute the energy dependence of  $\Gamma$  and  $\Delta$ . It all comes from the experimentally determined values of the elastic scattering nuclear phase shifts  $\delta_2$ .

# Two-step Reaction Simulation

Weight this with the probability that the  $\alpha$  escapes from the  $2^-$  resonance in the  $^{12}\text{C}$  nucleus (the penetrability for  $l=1$  (or 3) alpha particles) to get the probability distribution for populating the  $^8\text{Be}^*$  first excited state which subsequently decays into **two secondary  $\alpha$  particles having equal energies in the  $^8\text{Be}^*$  frame.**

Randomly distribute the primary and secondary  $\alpha$ 's in the center-of-mass frame while preserving proper kinematics and boosts back to the lab frame.

# Angular distributions

We need to know the form of the angular distribution of both the primary alphas, and of the secondary ones wrt. to the axis defined by the primary alphas.

These can be calculated using the  $Z_{\text{bar}}$  formalism of Blatt and Biedenharn.

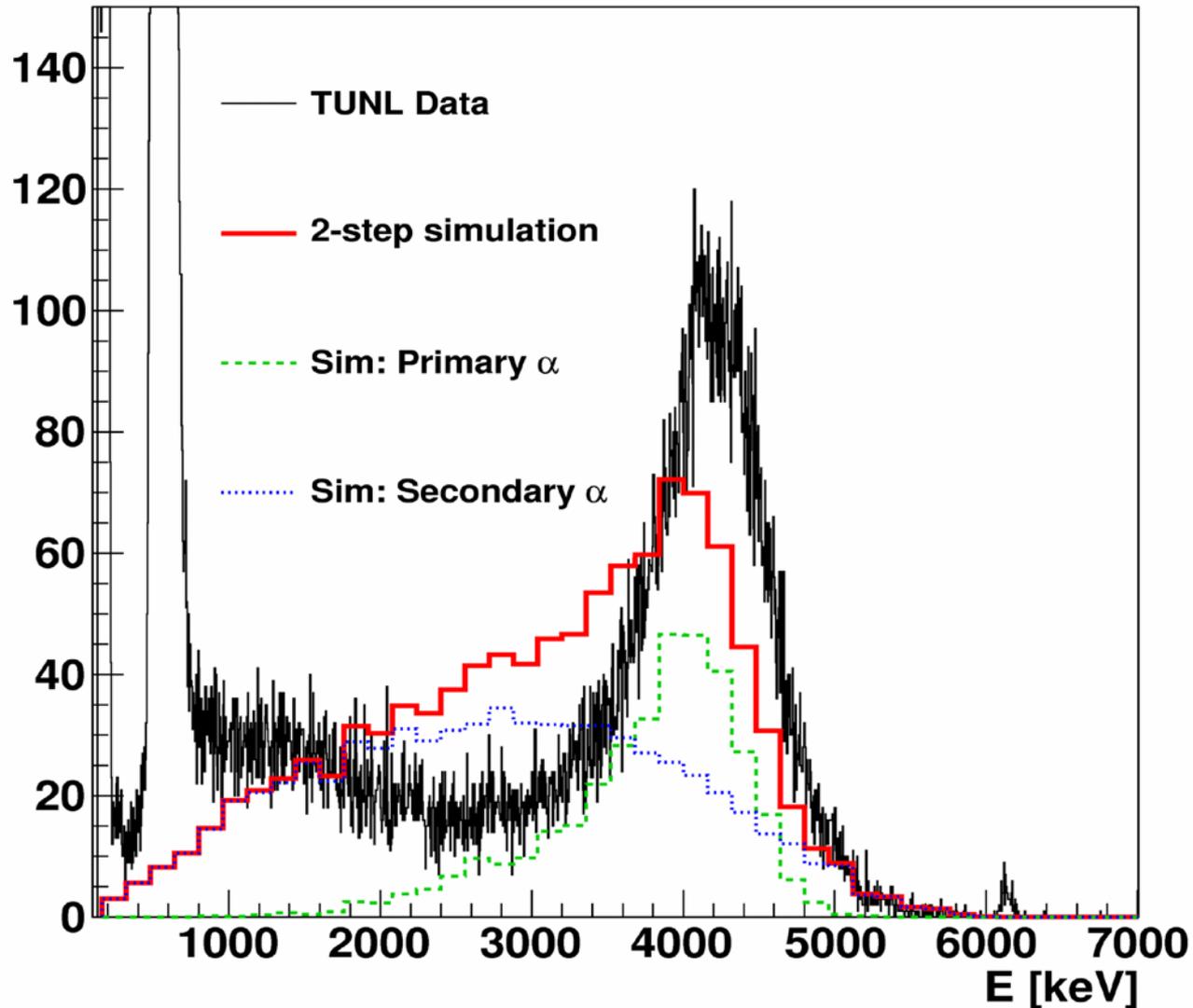
# For the $2^-$ state at 0.675 MeV

For the  $2^-$  (un-natural parity) state at 0.675 MeV, we have  $l=0$  and  $l'=1$  (according to Becker et al.). This immediately leads to an isotropic angular distribution for outgoing  $\alpha$ 's, as observed experimentally.

The distribution of the secondary  $\alpha$ 's wrt. the internal primary axis is found to be  $\sigma(\theta) = 1 - P_2(\cos\theta)$ , which is proportional to  $\sin^2\theta$ .

This is when the fun begins!

*Failure of the two-step model at the 2- resonance at 0.675 MeV ( $\Gamma = 300$  keV).*



# What went wrong?

When the  $2^-$  resonance in  $^{12}\text{C}$  decays to the first excited  $2^+$  state of  $^8\text{Be}$ , the *primary  $\alpha$ -particle can have orbital angular momentum of 1 or 3 units.*

*The 1987 work assumed the value of 1. We tried 3!*

To change to  $l=3$  outgoing primary alpha particles requires two changes in our simulation

1. Replace the  $l=1$  penetrabilities by  $l=3$  ones when generating the line shape of the  ${}^8\text{Be } 2^+$  state.
2. Use the angular distribution for the secondary alpha particles when the primary alphas have  $l=3$ .

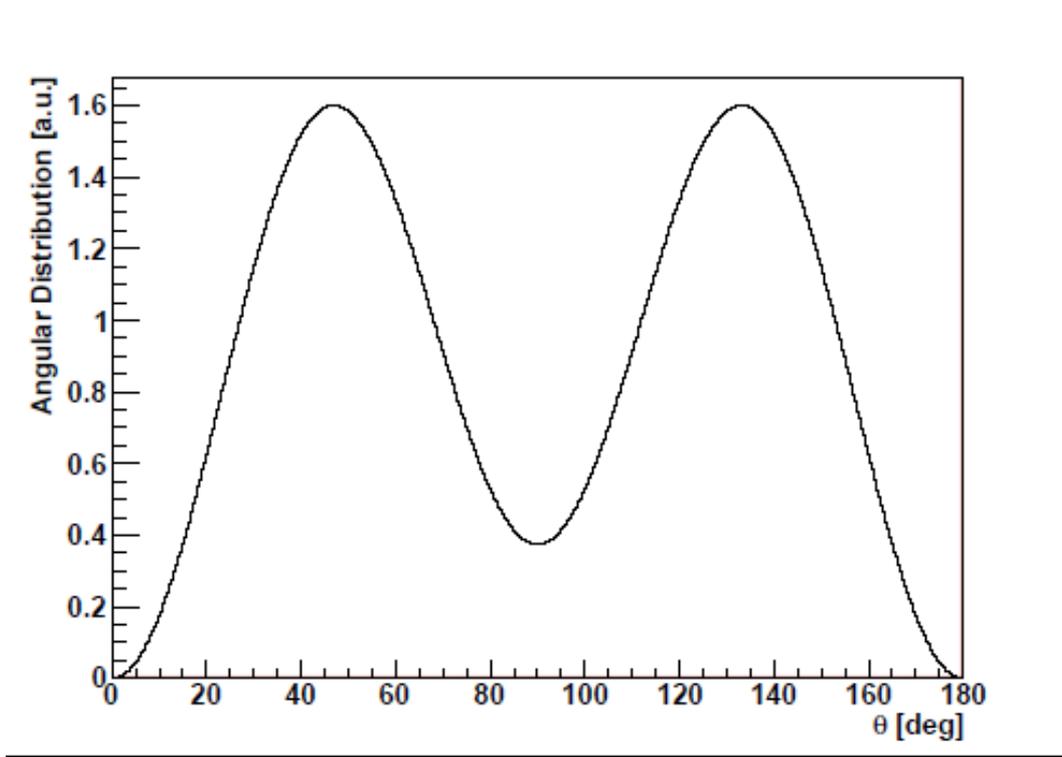
This is obtained by considering

$$(S=2^- + l=3 \rightarrow 2^+ \rightarrow S'=0 + l'=2)$$

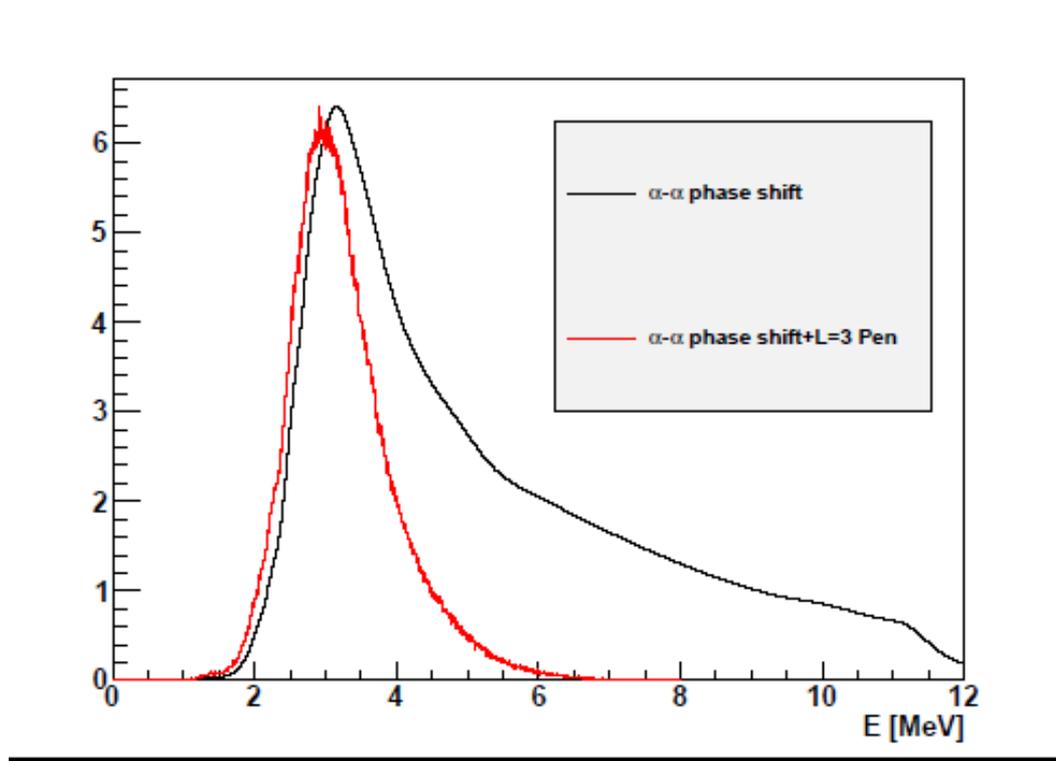
From  $Z_{\text{bar}}(3232;2L) Z_{\text{bar}}(2222;0L) P_L(\cos\theta)$

We obtain:  $\sigma(\theta) = C [1.0 + 2/7 P_2(\cos\theta) - 9/7 P_4(\cos\theta)]$

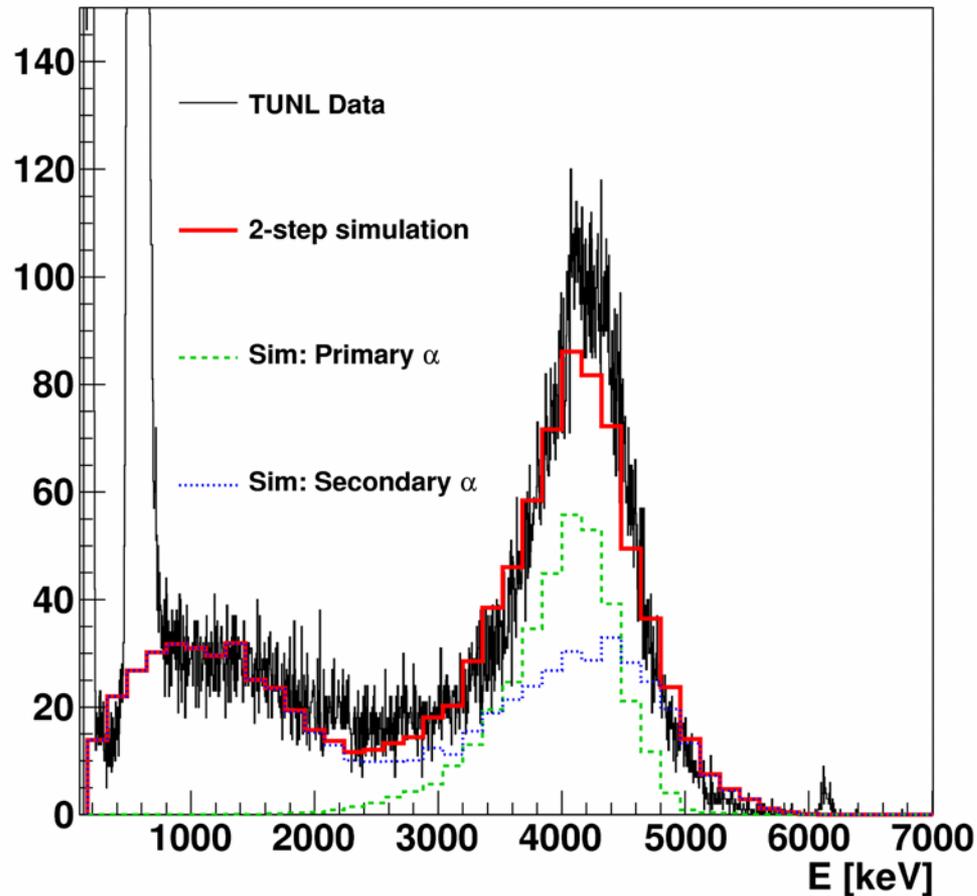
Which looks like:



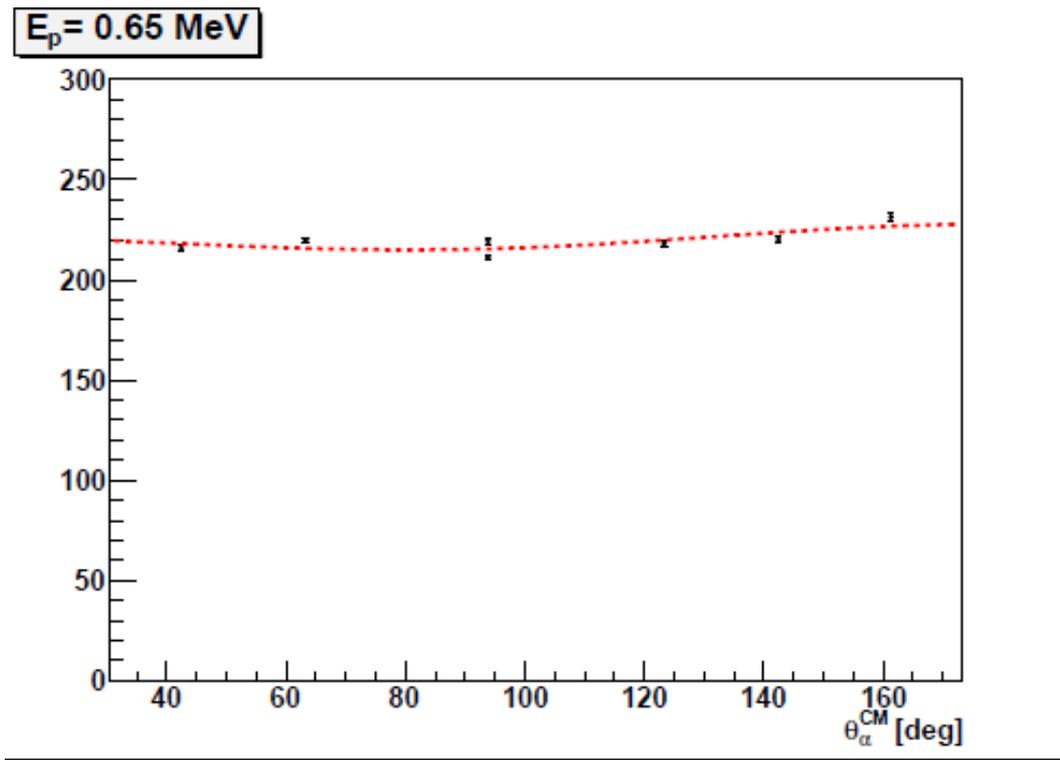
The effect of the  $l=3$  penetrability on the resonance line shape: a much narrower peak!  
(down from 1.5 to  $\sim 0.7$  MeV!!!)



# ***Success of the two step model with $l=3$ primary alpha particles***



And the angular distribution remains isotropic as observed.



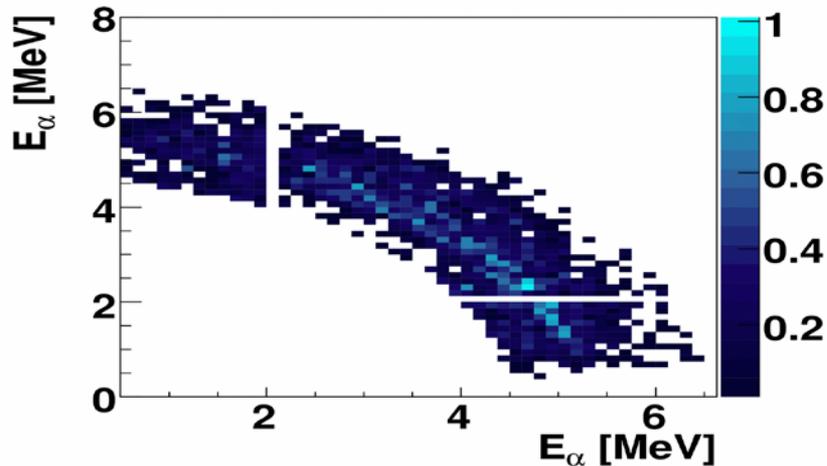
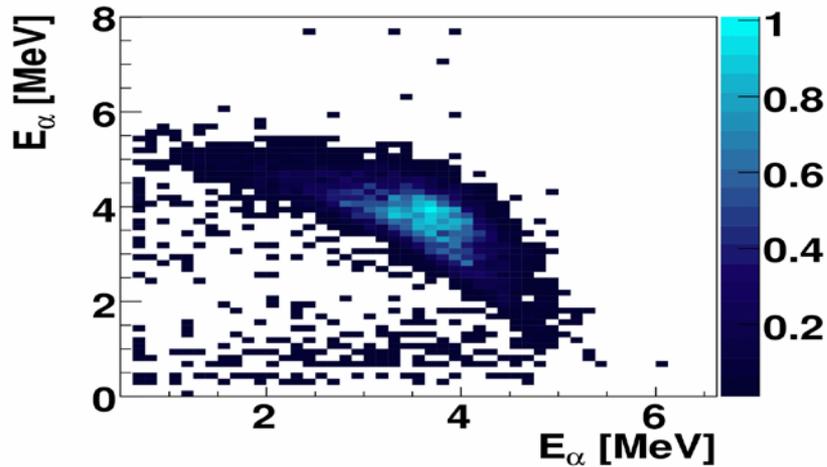
# What can additional experiments tell us?

We performed a coincidence experiment using position sensitive solid state detectors to increase the solid angle and provide data over an extended angular range.

Two detectors which each subtended an angular range of  $\pm 8^\circ$  were employed. Measured angular resolution was 0.2 degrees.

The detectors were placed symmetrically on the left and right sides of the beam. An opening angle range of  $100^\circ - 180^\circ$  was scanned.

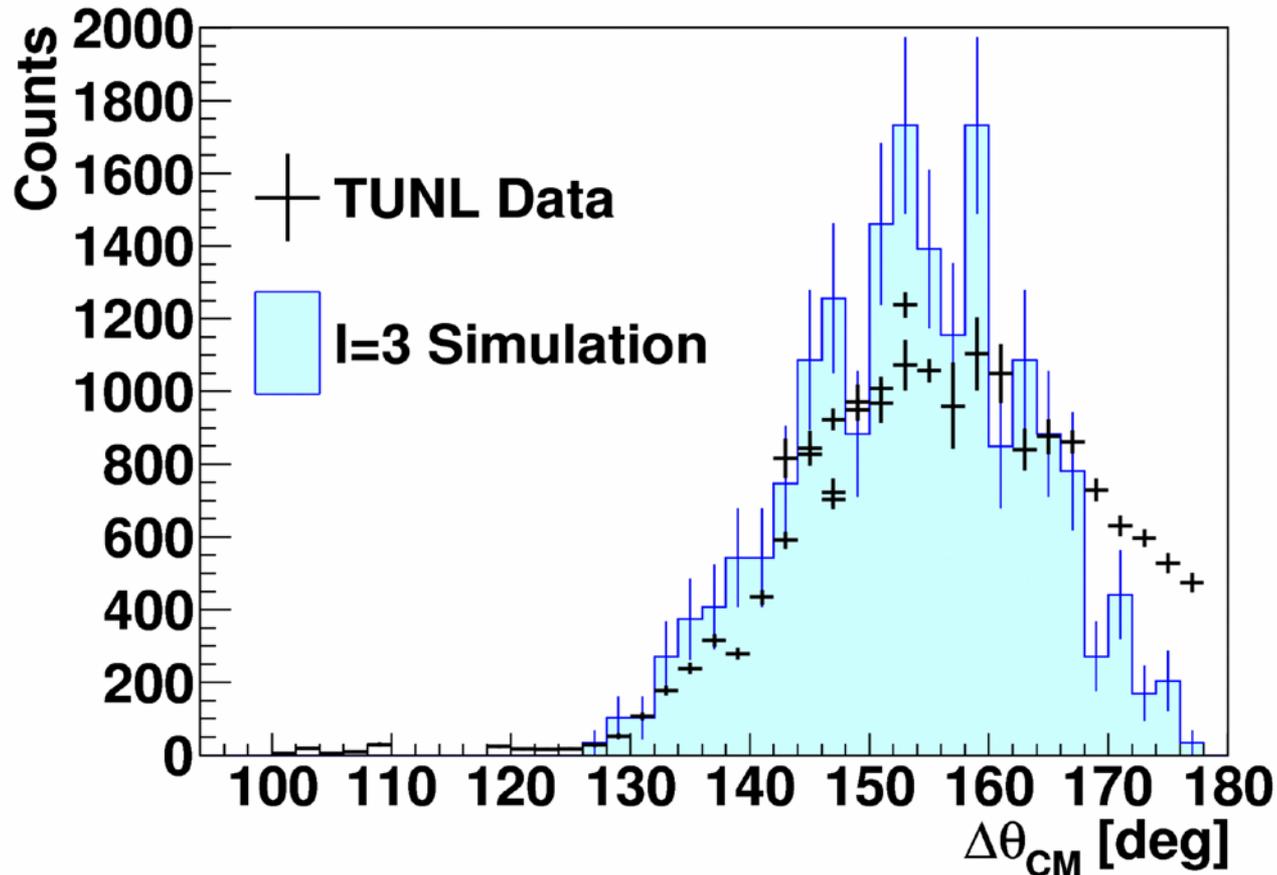
Coincidence spectra for the 0.675 MeV resonance (top) and the (3-) 2.64 MeV resonance (bottom) indicating two *high energy* alpha particles at 0.675 MeV. The lab  $\alpha$ - $\alpha$  opening angle is  $150^\circ$



# Testing the model

The simulation was asked to predict the coincidence counting rate as a function of opening angle. Coincidence data were taken for opening angles ranging from  $100^\circ$ -to- $180^\circ$ .

Both  $\alpha$ -particles were required to have energies greater than 3 MeV.  
Excellent agreement is observed.



# Conclusions

***We have found that a two-step sequential model with  $l=3$  primary  $\alpha$ -particles is required to describe the data at the 0.675 MeV  $2^-$  state.***

***The  $l=3$  assumption predicts the existence of two high-energy  $\alpha$ -particles at an opening angle centered at  $155^\circ$  –as originally proposed by Dee and Gilbert in 1936! Our coincidence measurements confirm this result.***

# *Huge impact on the reactor design!*

Our discovery of TWO high energy  $\alpha$ -particles is having a huge impact on the reactor design. They are much easier to extract and convert more efficiently into electricity.

Tri-Alpha Energy is thrilled!

(our results are published in **Physics Letters B** 696 (2011) 26-29 and **The Journal of Fusion Energy** (Springerlink.com), M.C. Spraker et al., 2011)

# Questions remain....

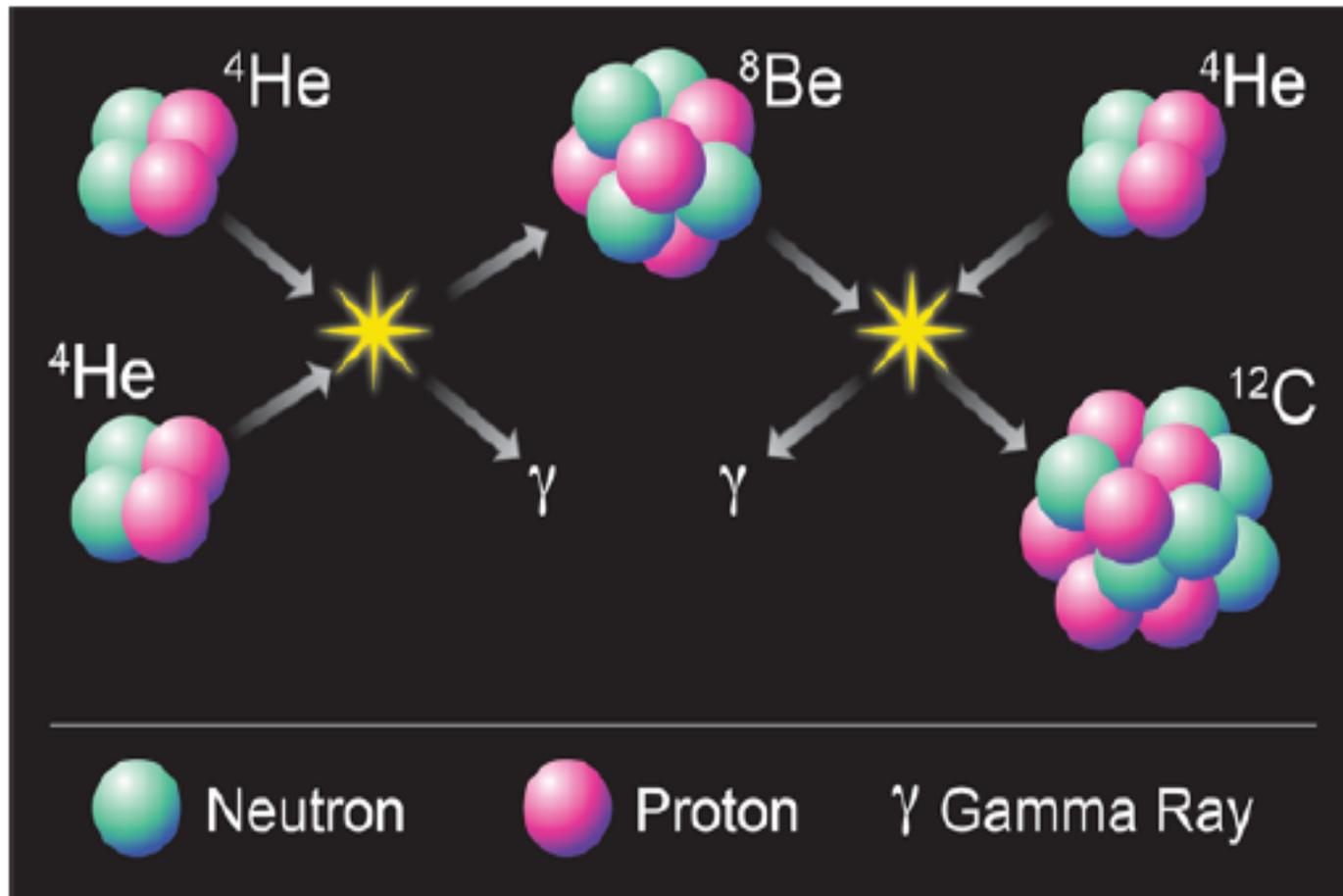
A challenge to theorists—

Why does the  $2^-$  resonance at 16.6 MeV decay via  $l=3$ ?

This resonance is assigned  $T=1$ , based upon its em decay properties. Also, the other members of the isospin triplet are observed in  $^{12}\text{B}$  and  $^{12}\text{N}$  (second excited states).

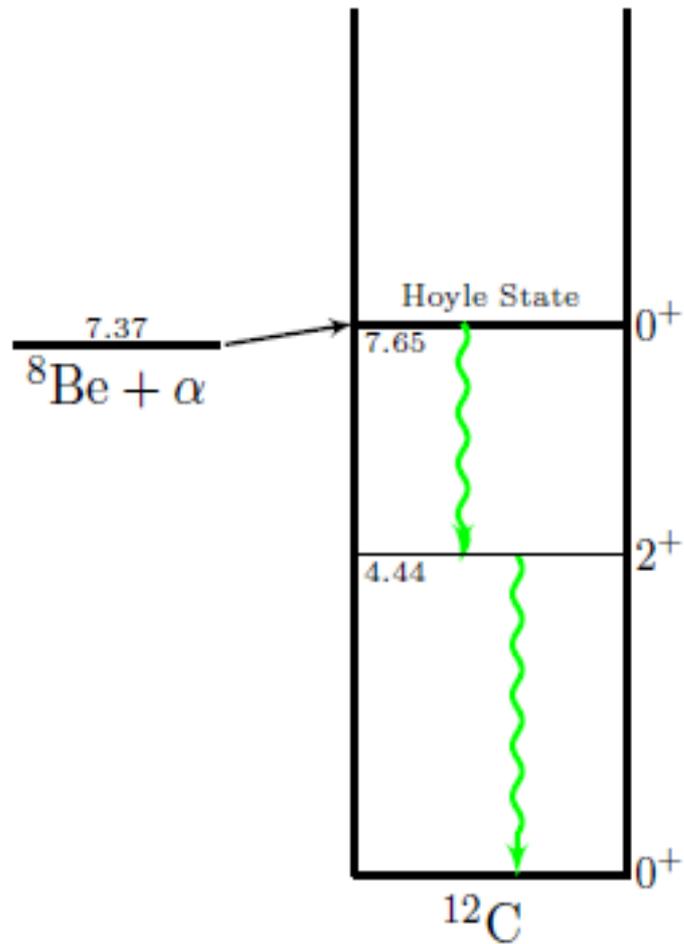
How does this  $T=1$  state decay into 3 alphas? Obviously there is strong isospin mixing.

## Triple- $\alpha$ Process:

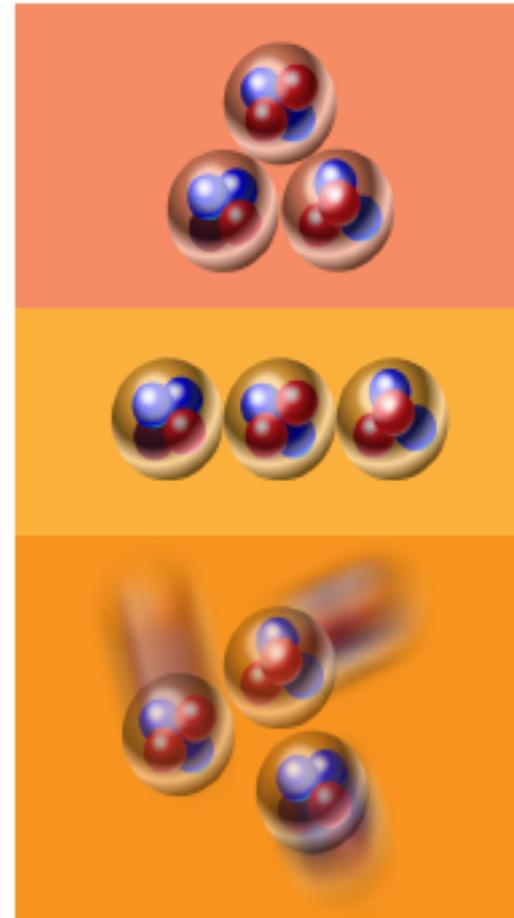
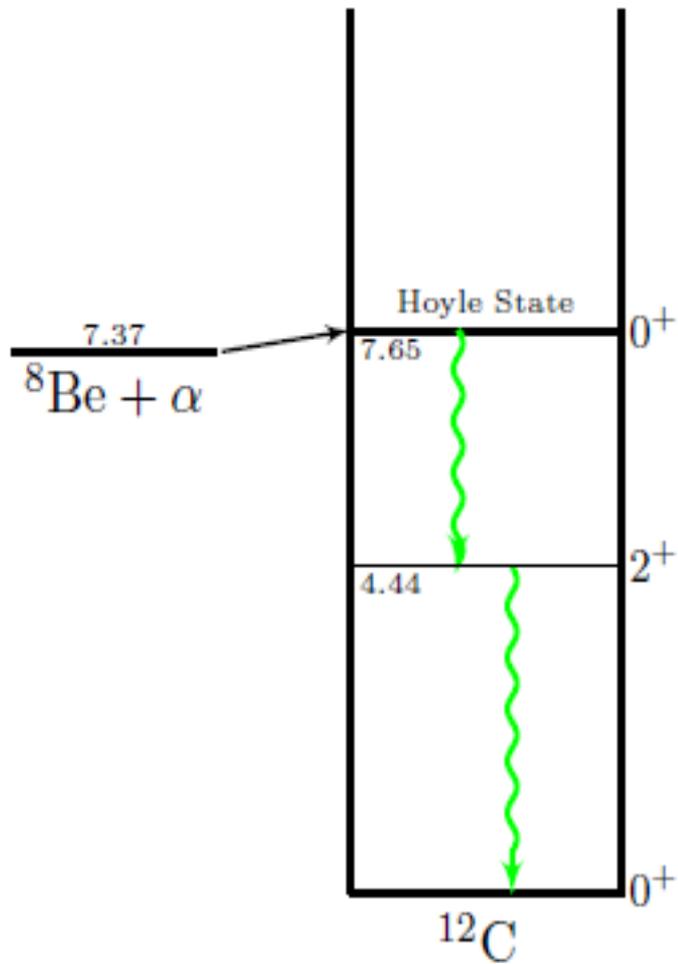


M. Hjorth-Jensen, Physics 4, 38 (2011)

# Hoyle State



# Hoyle State



Fynbo & Freer, Physics 4, 94 (2011)

More recently, Epelbaum et al. (Lattice EFT) find an obtuse triangular configuration.

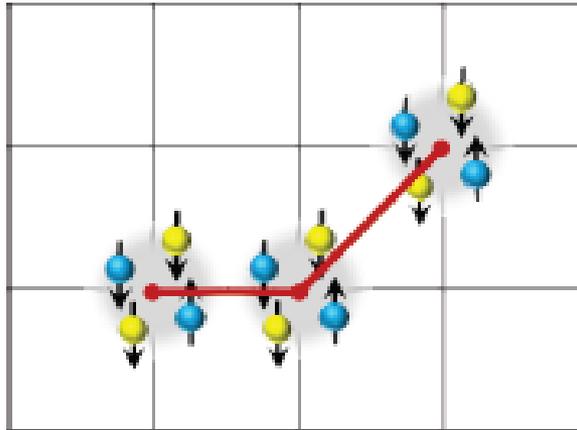
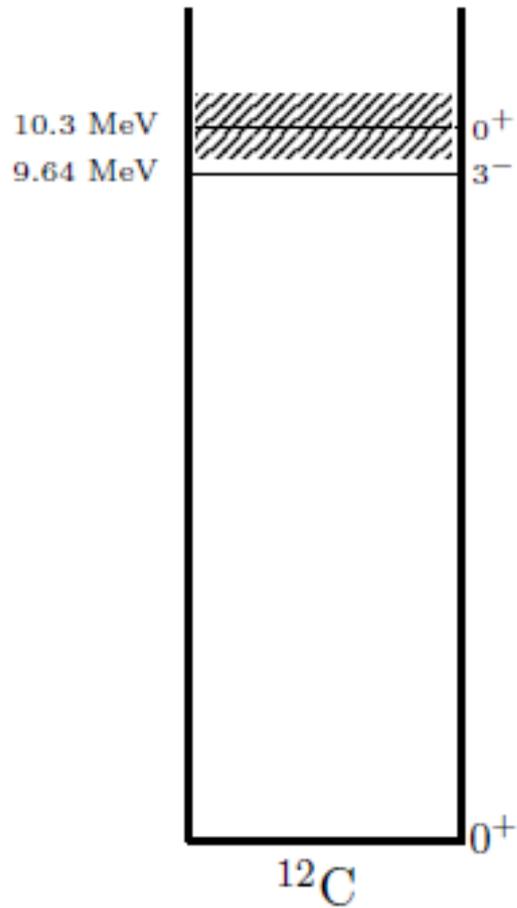
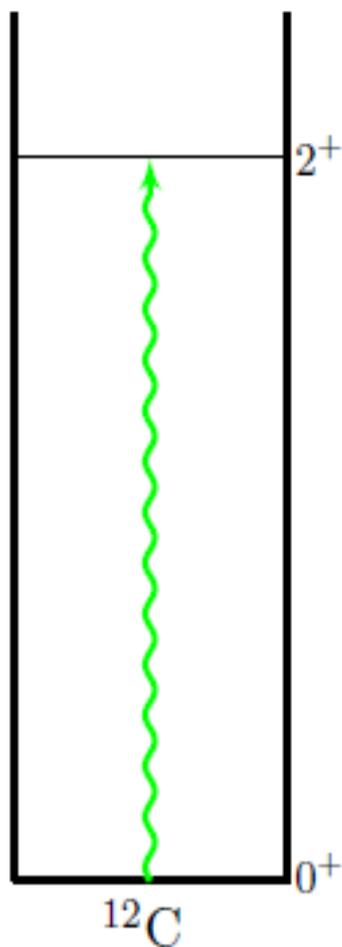


FIG. 3: This shows initial state  $F$ , a wavefunction consisting of three alpha clusters formed by Gaussian packets centered on the vertices of a bent-arm or obtuse triangular configuration. There are a total of 24 equivalent orientations of this configuration.

# $^{12}\text{C}(\gamma, \alpha)^8\text{Be}$

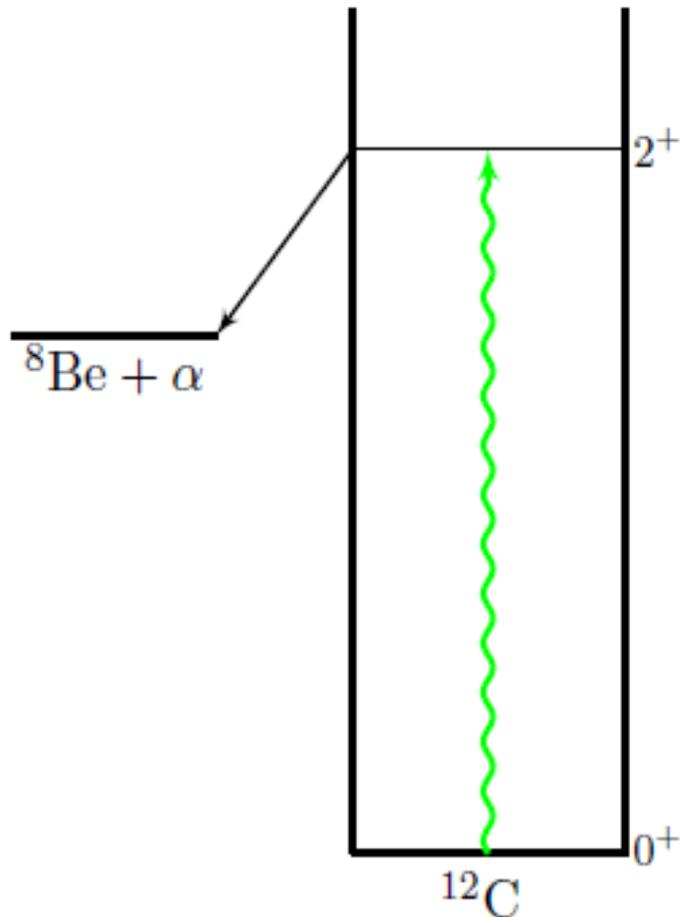


# $^{12}\text{C}(\gamma,\alpha)^8\text{Be}$

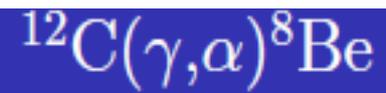


- $\gamma$ -ray beam will not excite  $0^+$  state and will excite  $3^-$  state with very small probability.

# $^{12}\text{C}(\gamma, \alpha)^8\text{Be}$



- $\gamma$ -ray beam will not excite  $0^+$  state and will excite  $3^-$  state with very small probability.
- Measuring  $\alpha$ -particle angular distribution gives  $J^\pi$  of state.



Measuring  $^{12}\text{C}(\gamma, \alpha)^8\text{Be}$  requires:

# $^{12}\text{C}(\gamma, \alpha)^8\text{Be}$

Measuring  $^{12}\text{C}(\gamma, \alpha)^8\text{Be}$  requires:

- Intense, monoenergetic  $\gamma$ -ray beam

# $^{12}\text{C}(\gamma, \alpha)^8\text{Be}$

Measuring  $^{12}\text{C}(\gamma, \alpha)^8\text{Be}$  requires:

- Intense, monoenergetic  $\gamma$ -ray beam
- Detector capable of measuring angular distributions of recoiling  $\alpha$ -particles with little or no background

# •HI $\gamma$ S

- High Intensity  $\gamma$ -ray Source  
(HI $\gamma$ S)

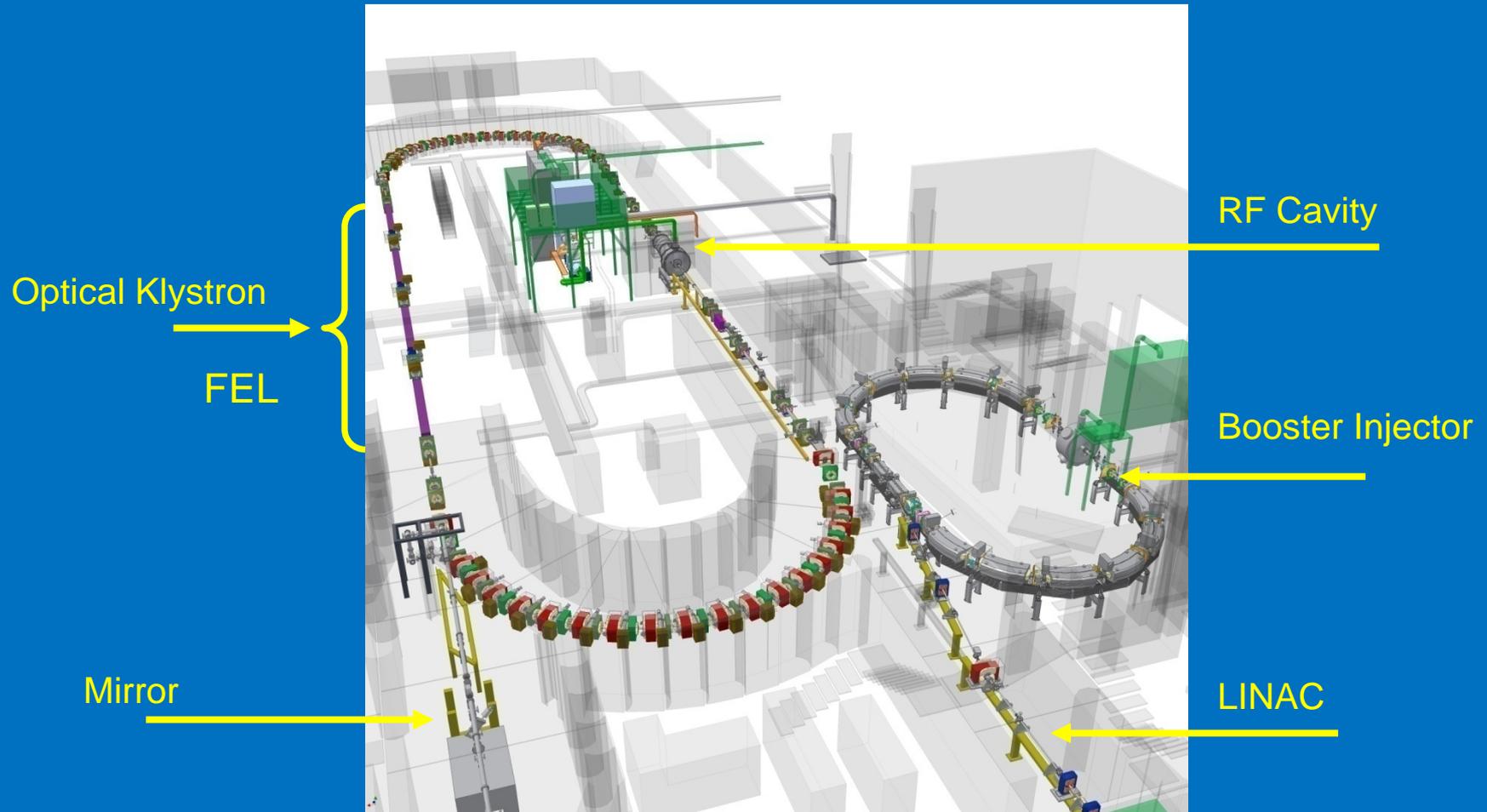
- Located at the Duke Free Electron  
Laser Laboratory  
Part of the Triangle Universities  
Nuclear Laboratory (TUNL)

- Intra-cavity Compton Back  
Scattering of FEL photons by  
electrons circulating in the Duke  
Storage Ring

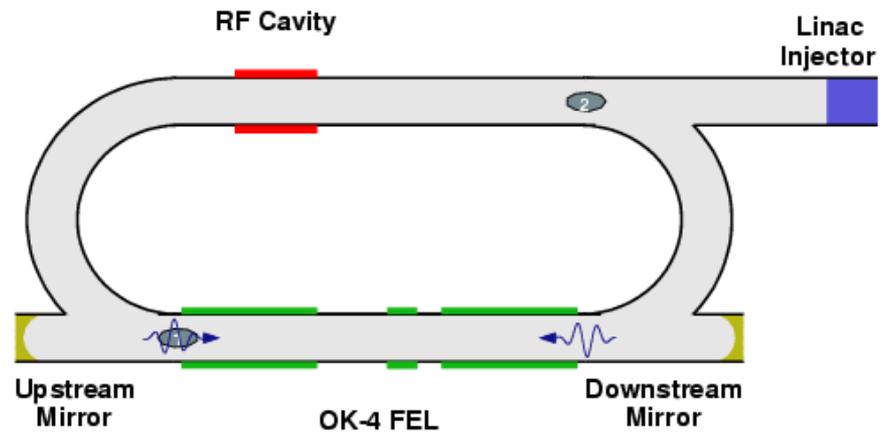


# HI $\gamma$ S $\gamma$ -ray beam generation

Provides circularly and linearly polarized, nearly **monoenergetic**  $\gamma$ -rays from 2 to 100 MeV  
Utilizes Compton backscattering to generate  $\gamma$ -rays



## Two Bunch Mode



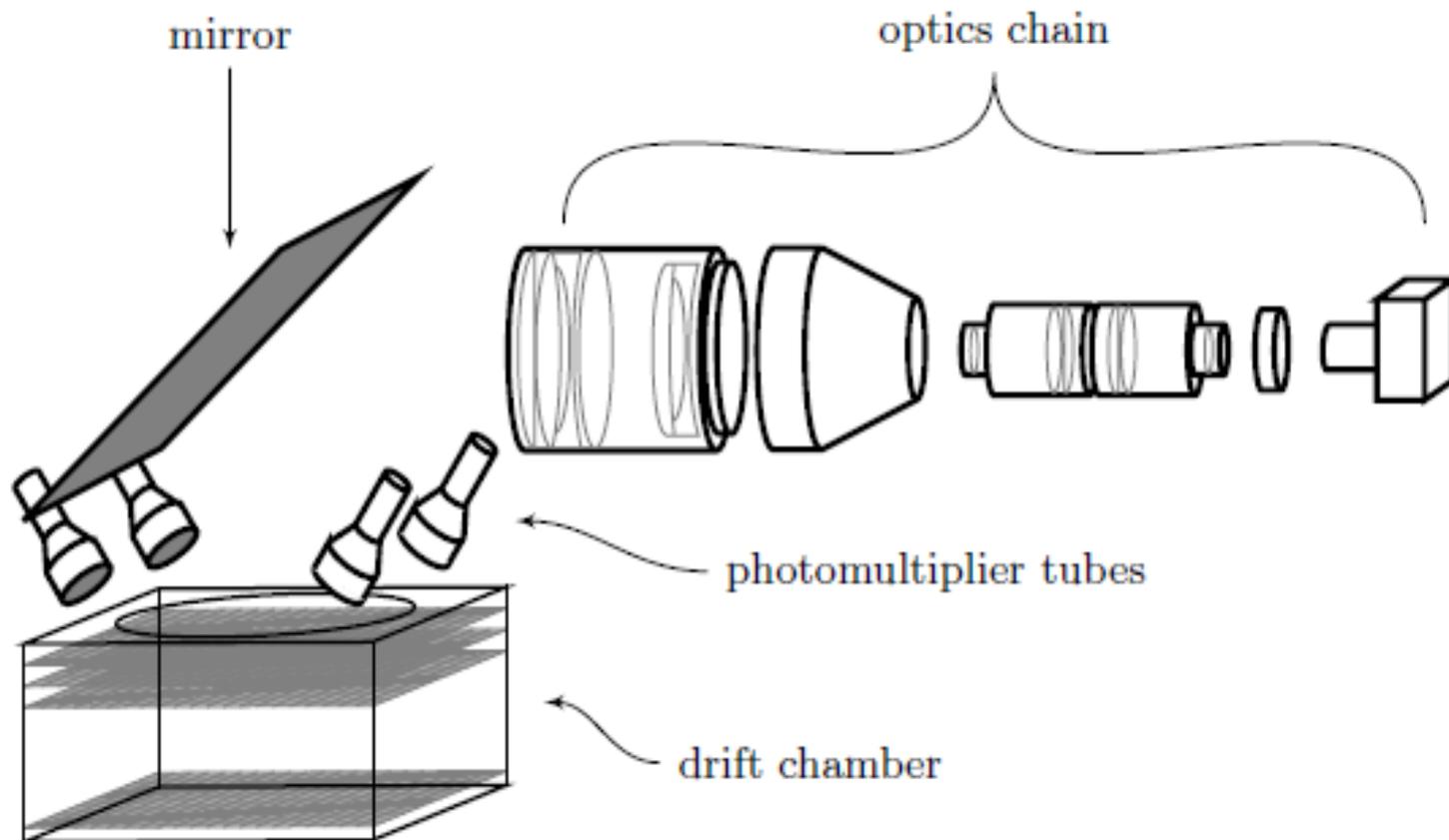
Created by Brent Perdue, 2005

## ***Some typical beam intensities***

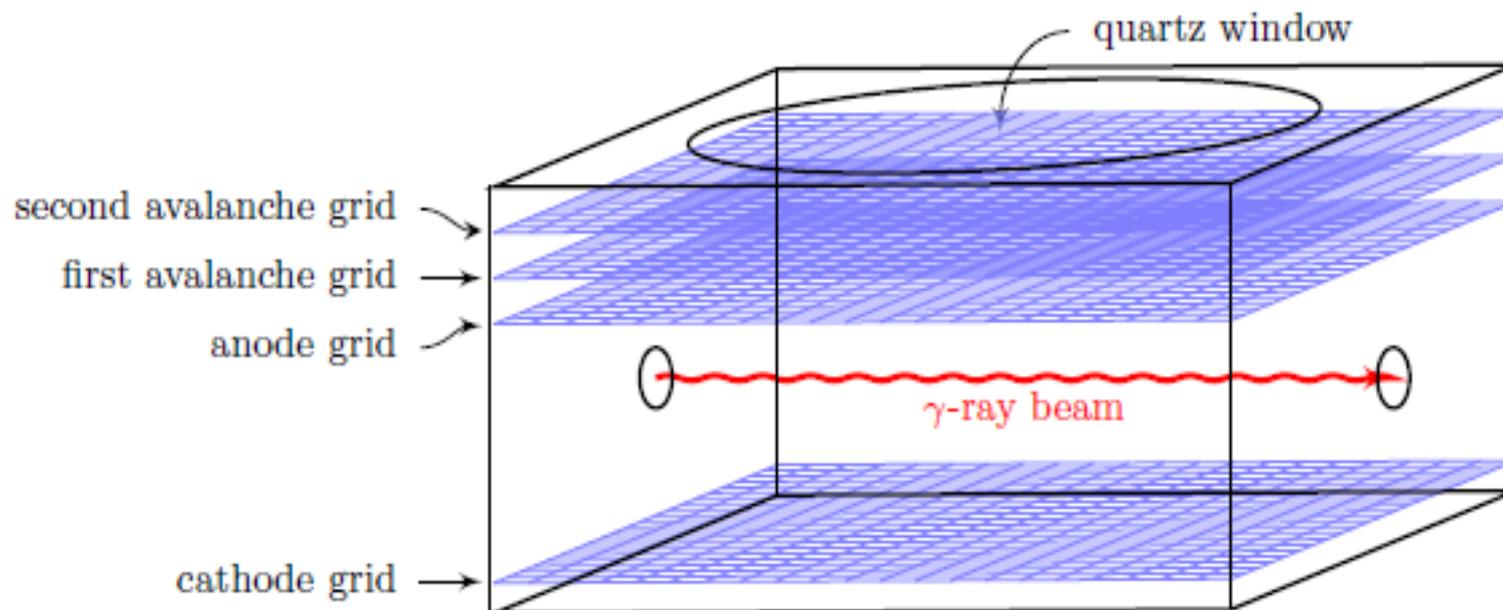
<u><math>E_\gamma</math> (MeV)</u>	<u>Beam on target (<math>\Delta E/E = 3\%</math>)</u>
1 - 2	$2 \times 10^7 \text{ } \gamma/\text{s}$
8 - 16	$2 \times 10^8$ (total flux of $2 \times 10^9$ )
20 - 45	$8 \times 10^6$
50 - 100	$4 \times 10^6$ (will increase by x3-5 in 2012)
→160	$1.2 \times 10^7$ (by 2015)

Show  $H\gamma S$  animation.

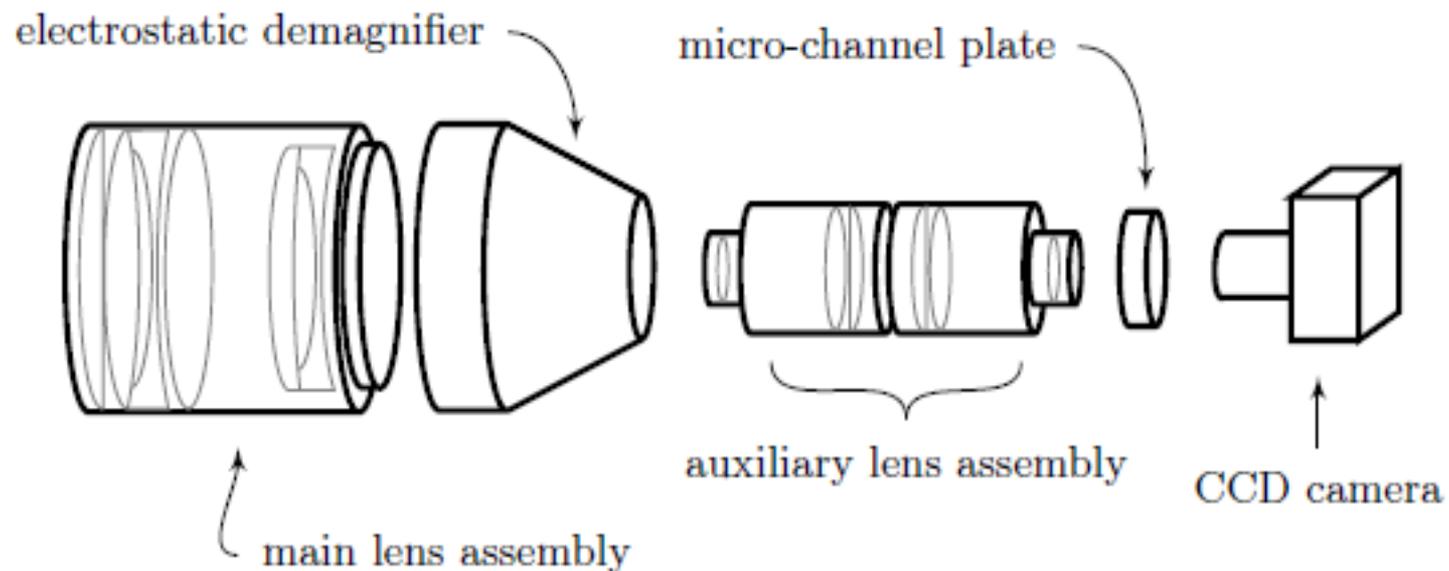
# HI $\gamma$ S O-TPC



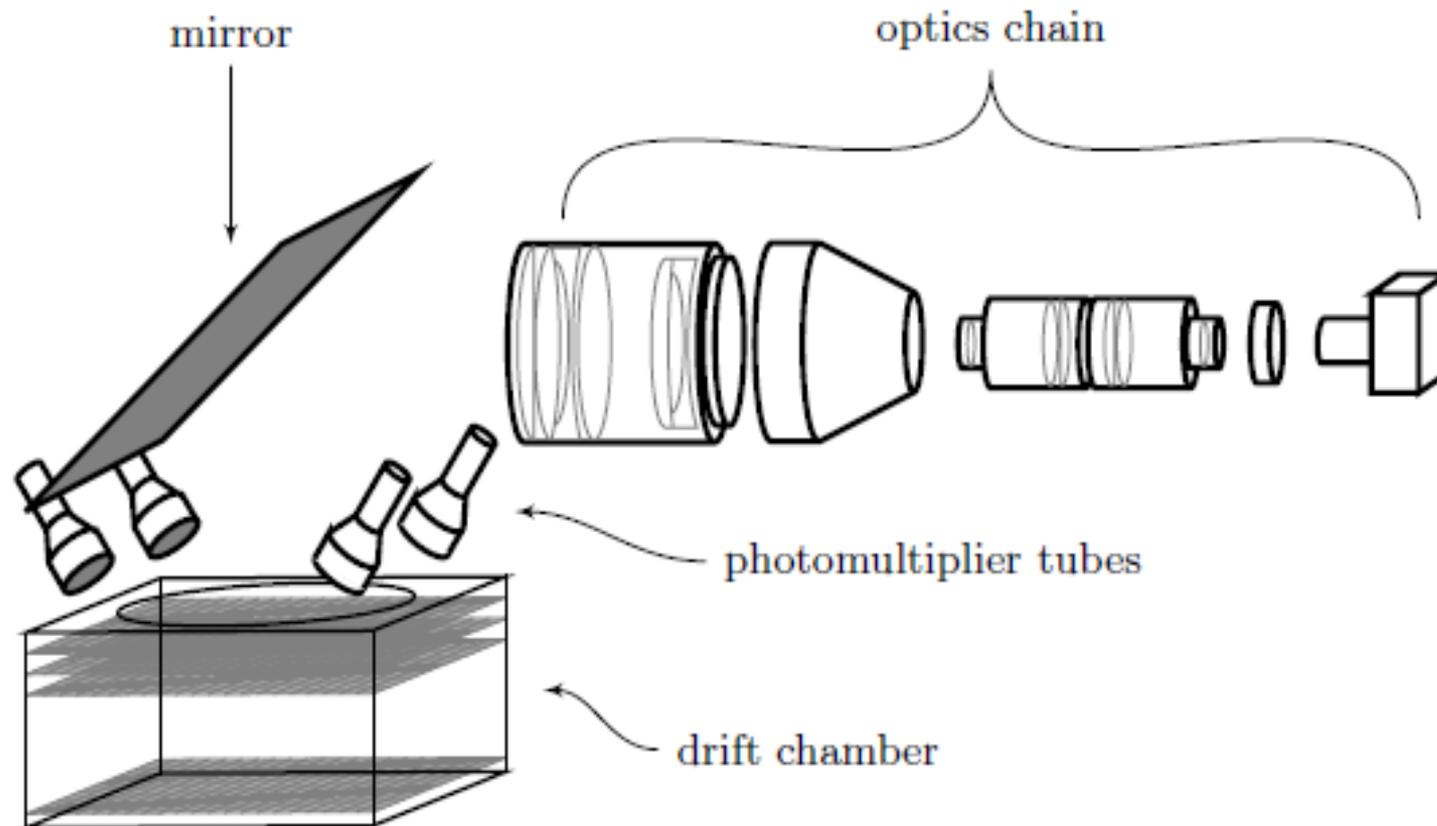
## Drift Chamber



## Optics Chain

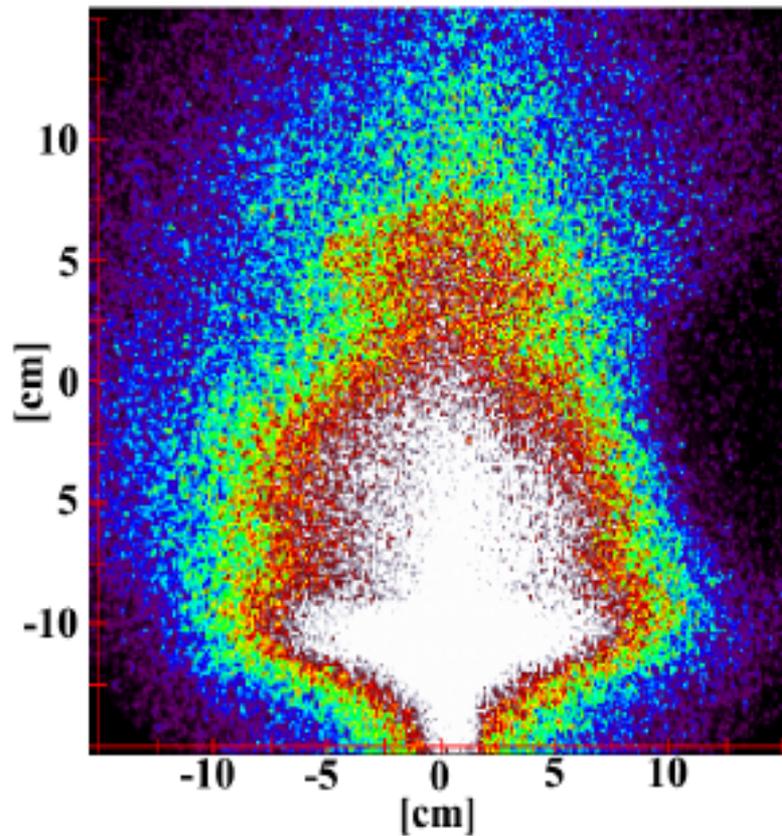


# HI $\gamma$ S O-TPC

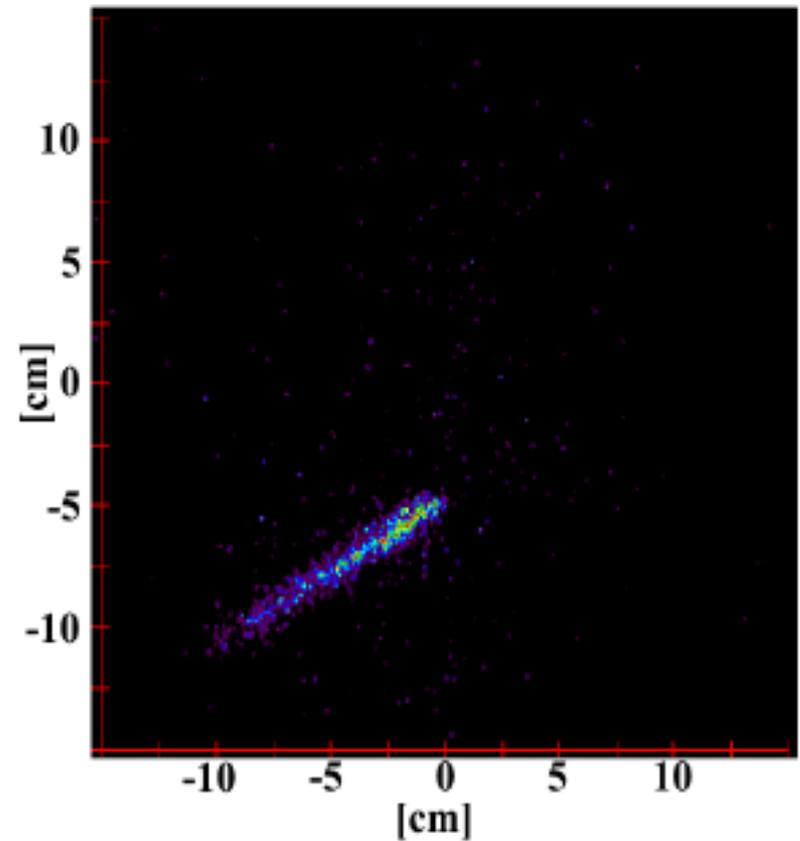


# Background Rejection

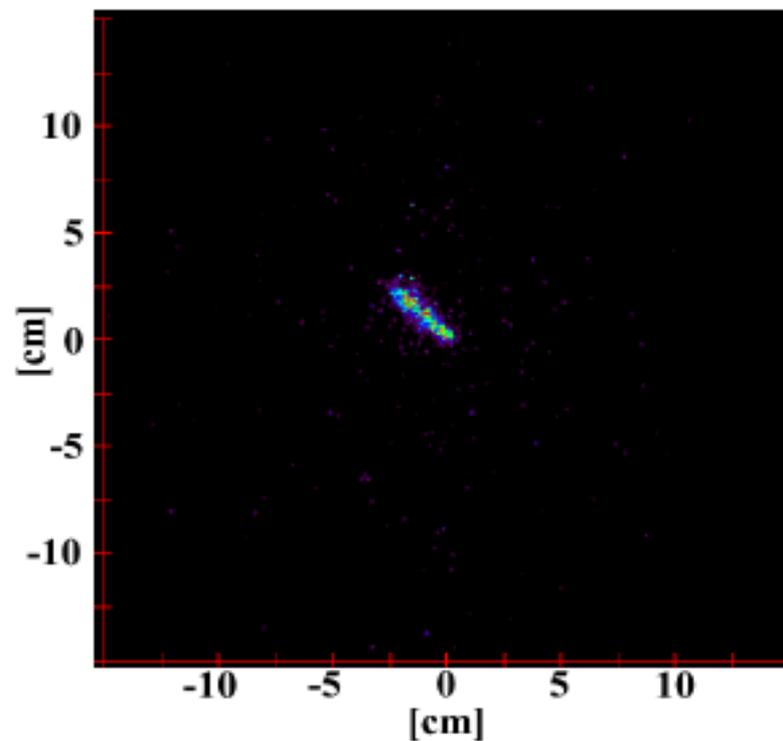
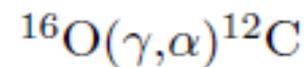
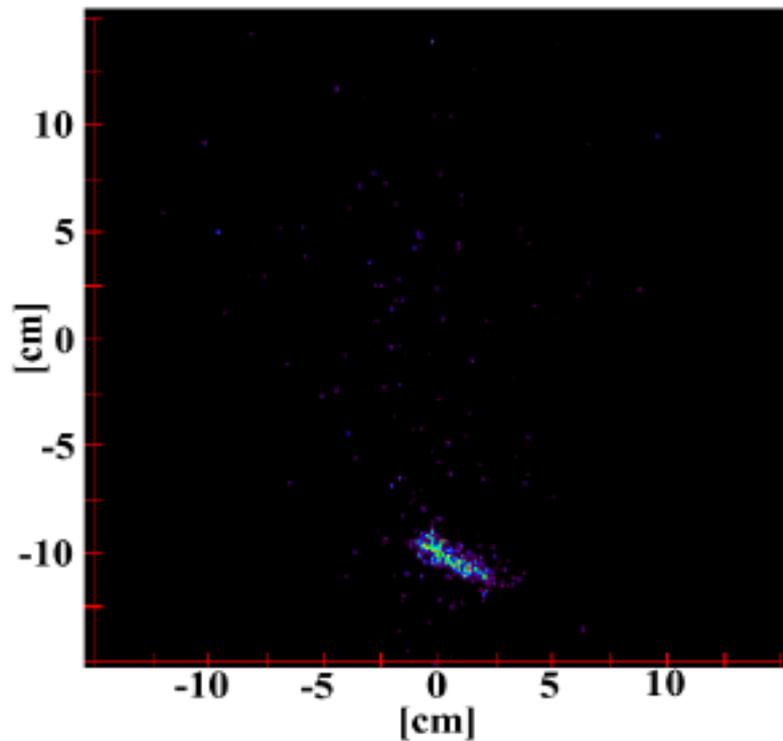
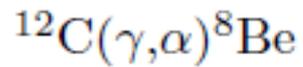
Spark



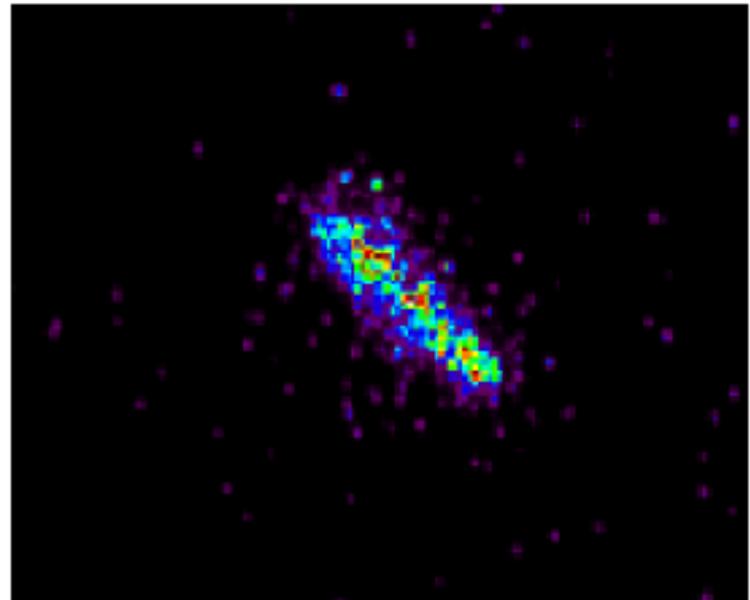
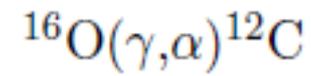
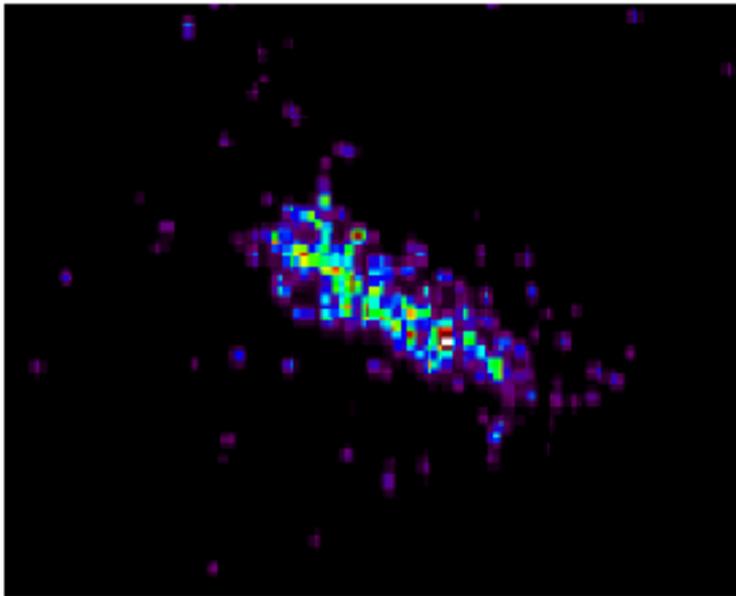
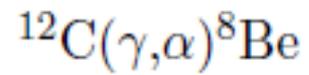
Cosmic



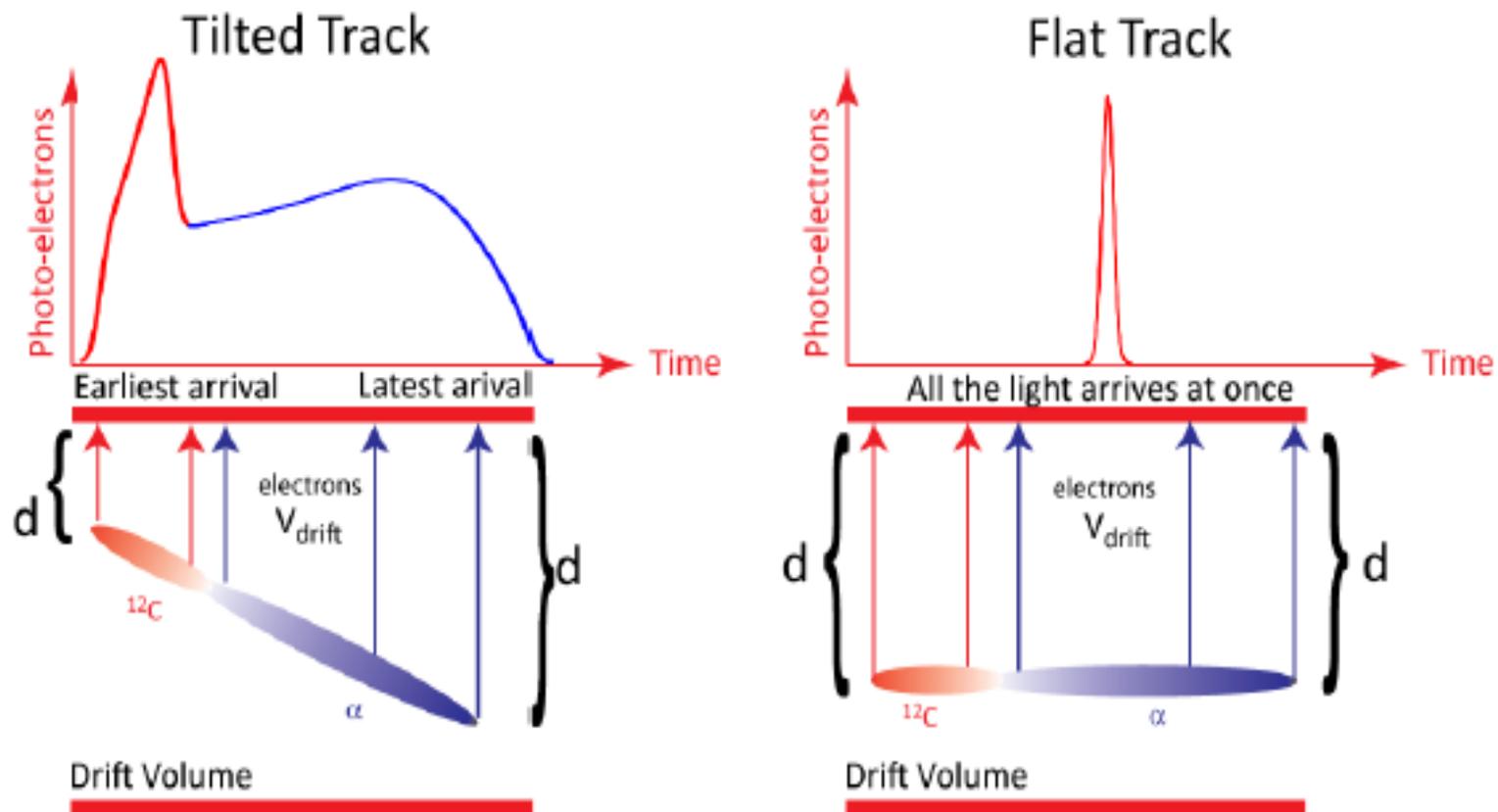
# Event Identification



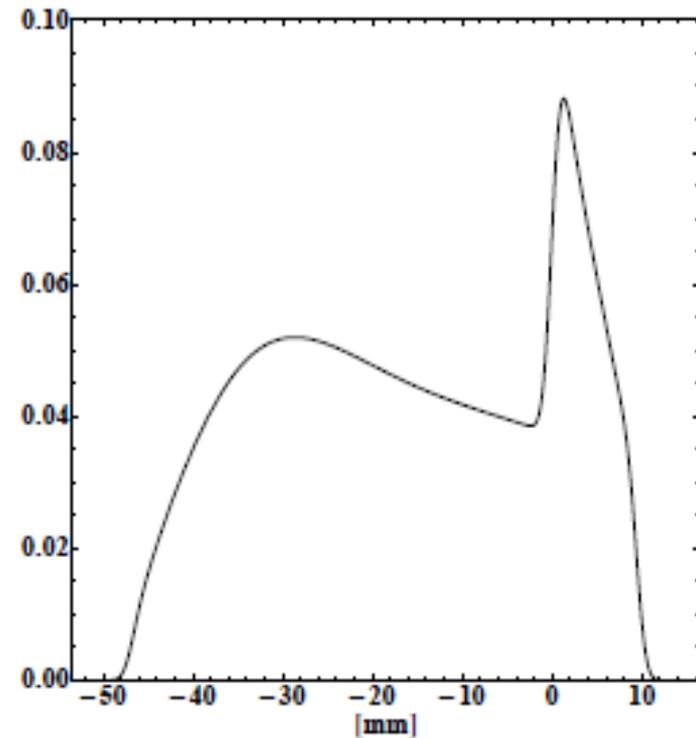
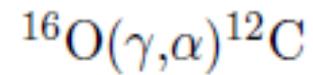
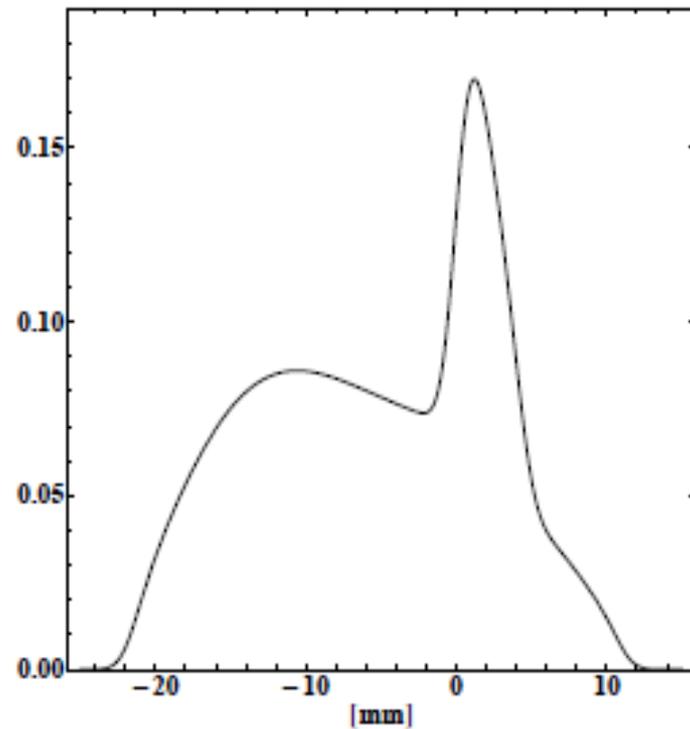
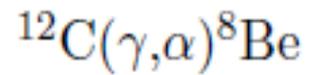
# Event Identification



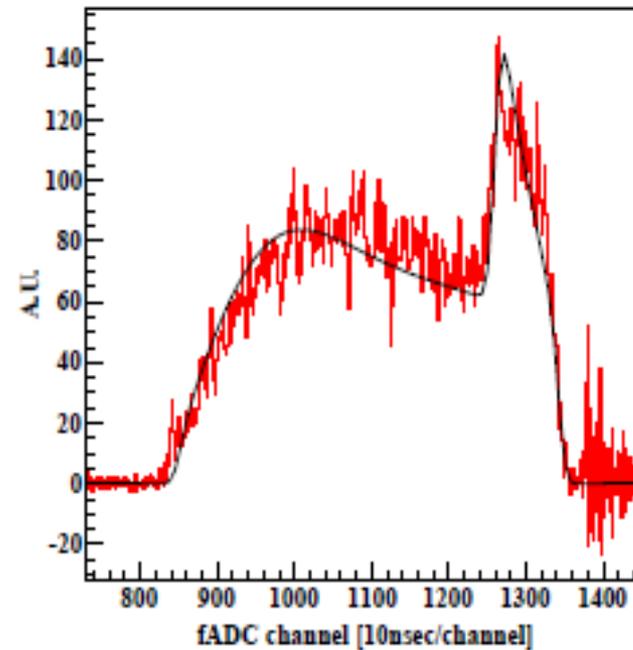
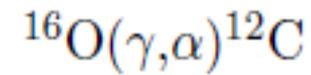
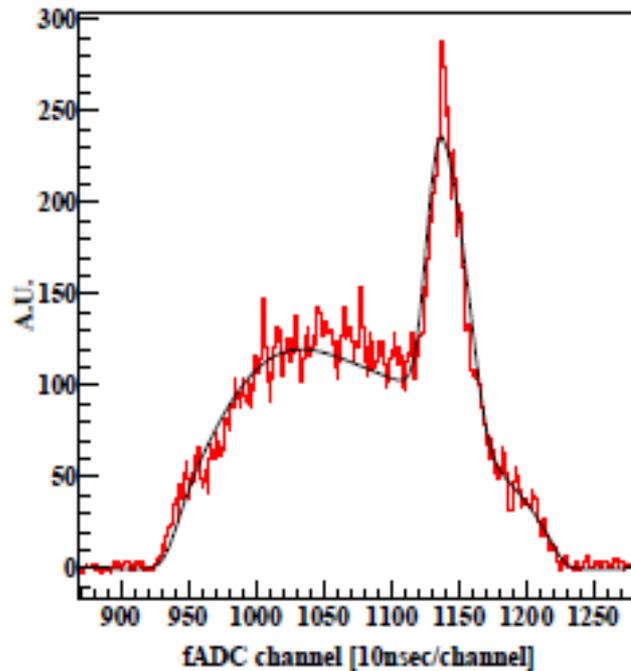
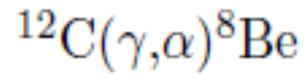
# Time Projection



# Time Projection



# Time Projection



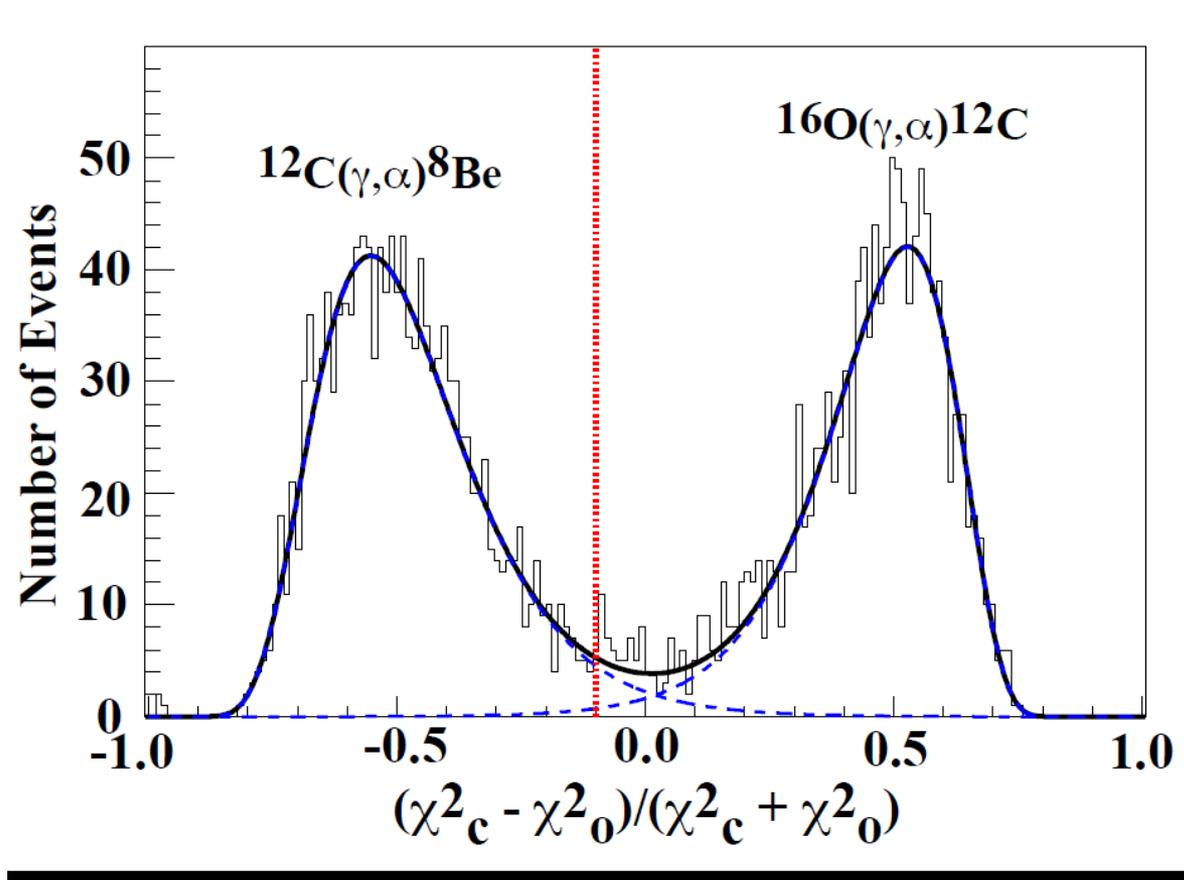
# Time Projection

$^{12}\text{C}(\gamma,\alpha)^8\text{Be}$  event,  
fit assuming  $^{12}\text{C}(\gamma,\alpha)^8\text{Be}$

$^{12}\text{C}(\gamma,\alpha)^8\text{Be}$  event,  
fit assuming  $^{16}\text{O}(\gamma,\alpha)^{12}\text{C}$

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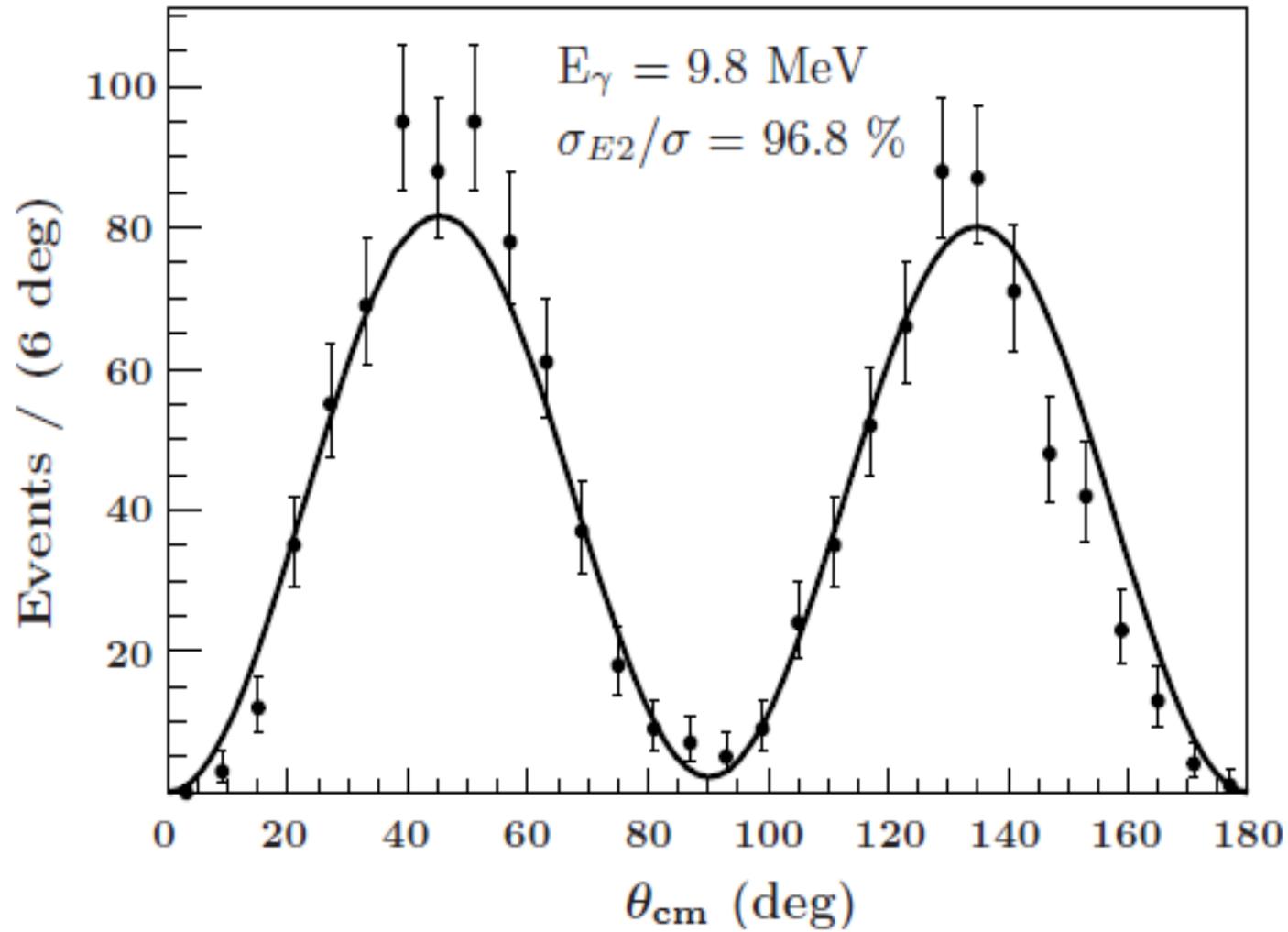
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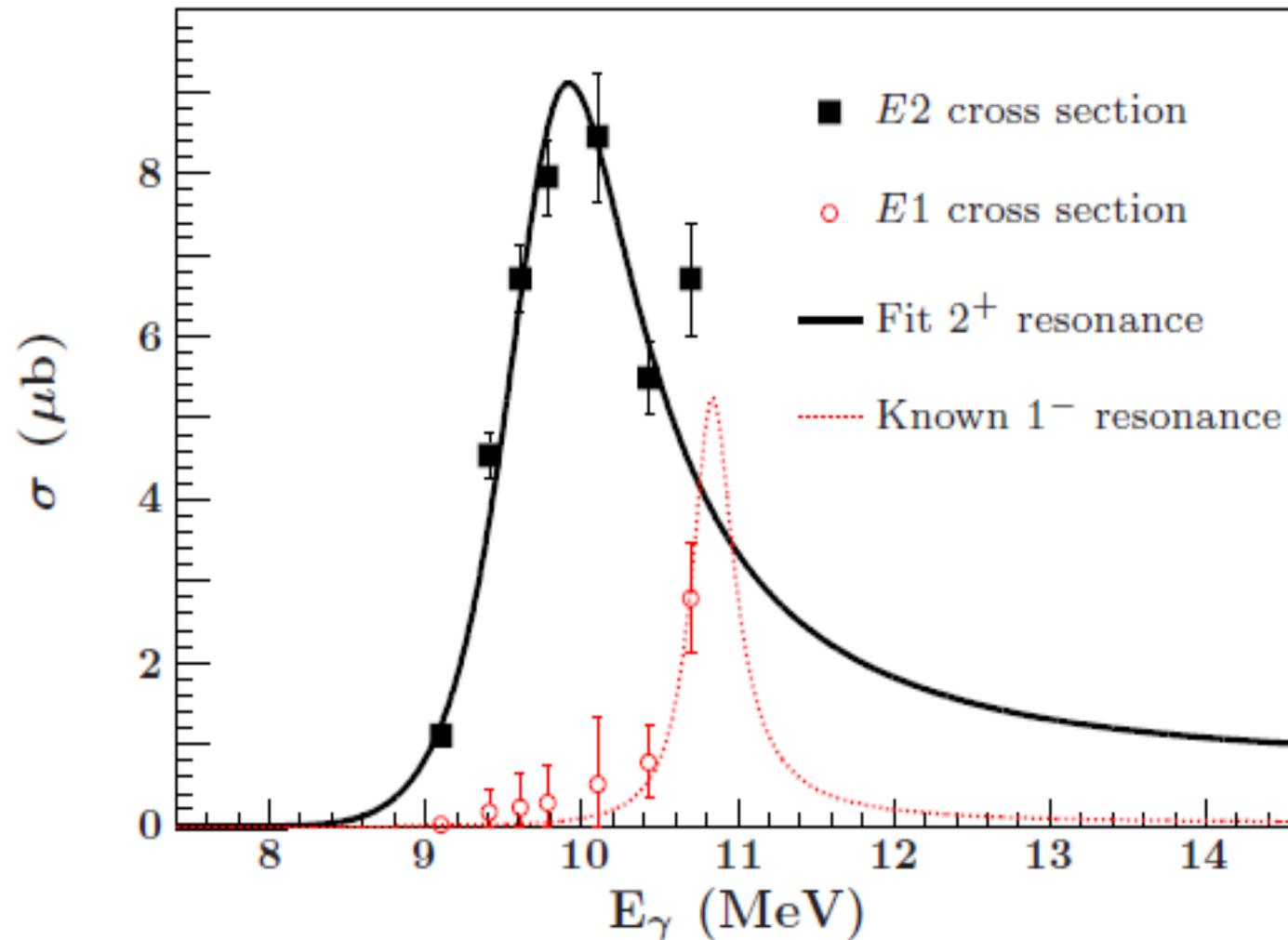
- Since angular information is available for each event, Unbinned Maximum Likelihood fits were used.

$$\mathcal{L}(|E1|, |E2|, \phi_{12}) = \prod_{i=1}^n W(\theta_i)$$

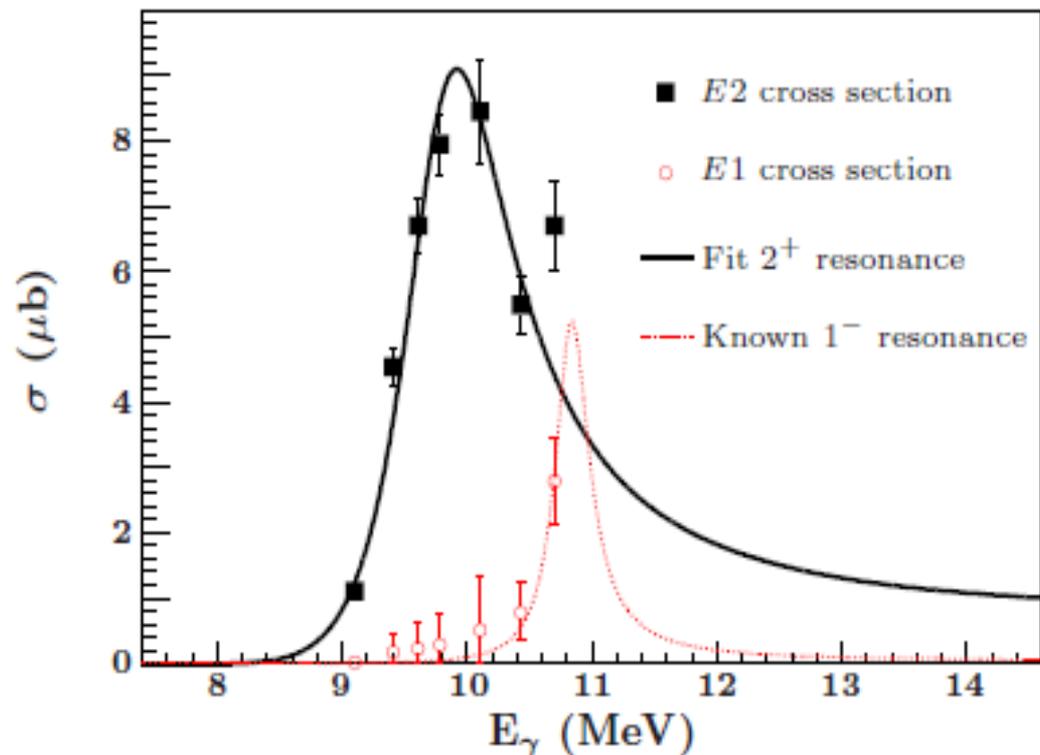
# Angular Distributions



# Cross Section



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$E_{\text{res}}$ (MeV)	$\Gamma_{\alpha}(\text{res})$ (keV)	$\Gamma_{\gamma_0}(\text{res})$ (meV)	$B(E2 : 2_2^+ \rightarrow 0_1^+)$ ( $e^2\text{fm}^4$ )
10.03(11)	800(130)	60(10)	0.73(13)

# Phase

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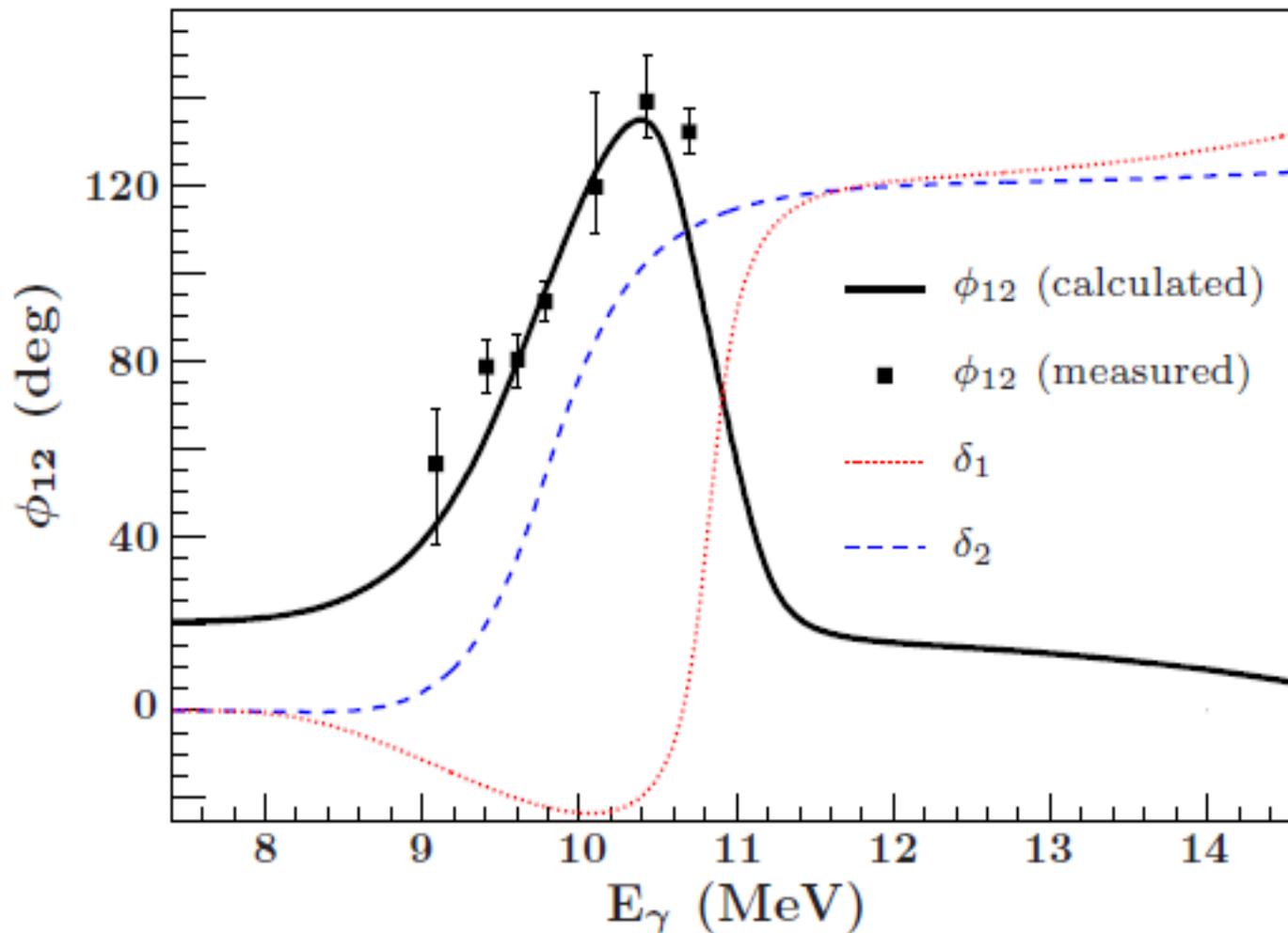
Nuclear phase shifts:

$$\delta_\ell = \arctan\left(\frac{\Gamma_\ell}{2(E_{r\ell} - \Delta_\ell - E_{cm})}\right) - \phi_\ell$$

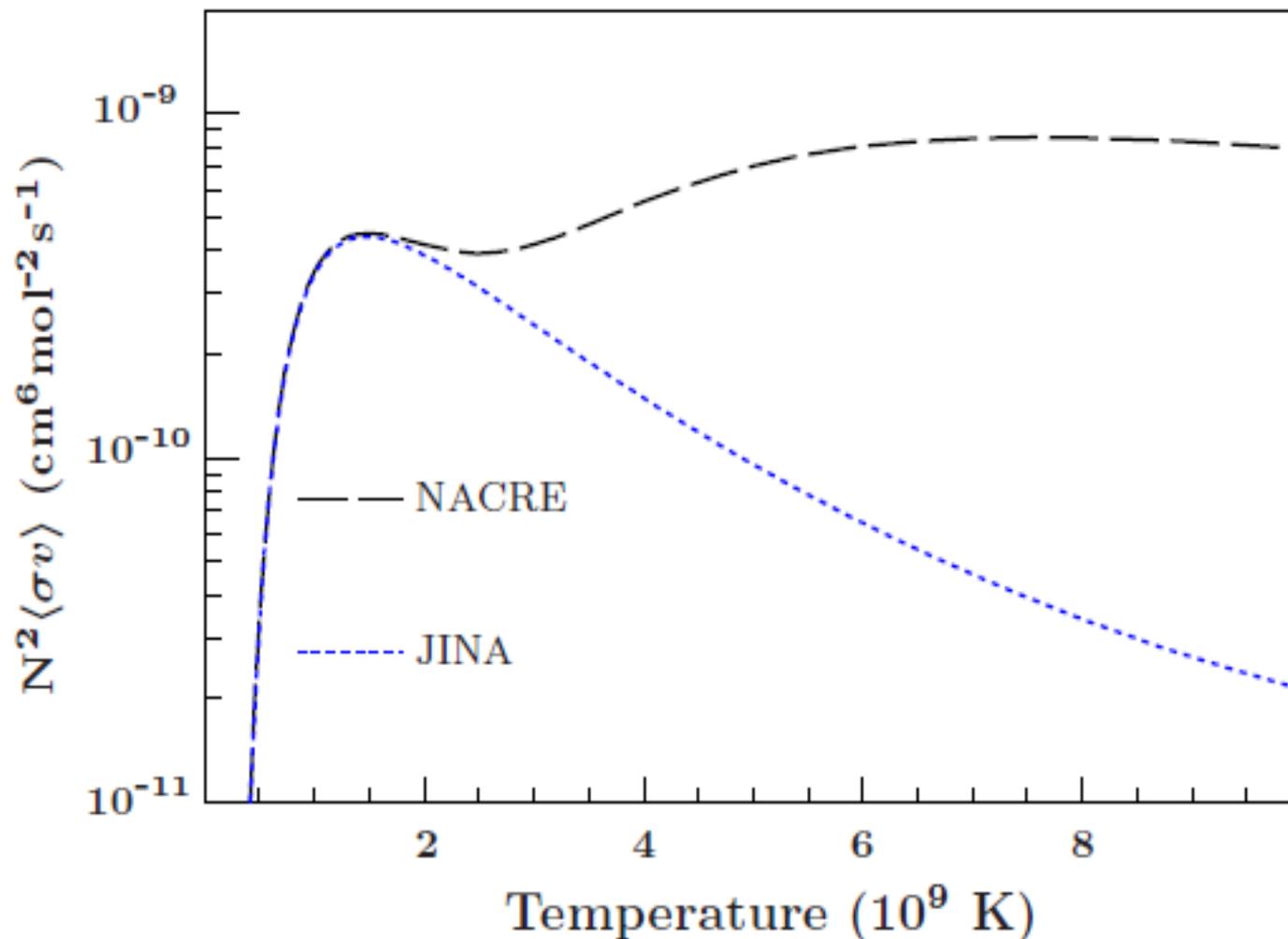
Hard sphere scattering phase shift:

$$\phi_\ell = \arctan\left[\frac{F_\ell}{G_\ell}\right]_{r=a}$$

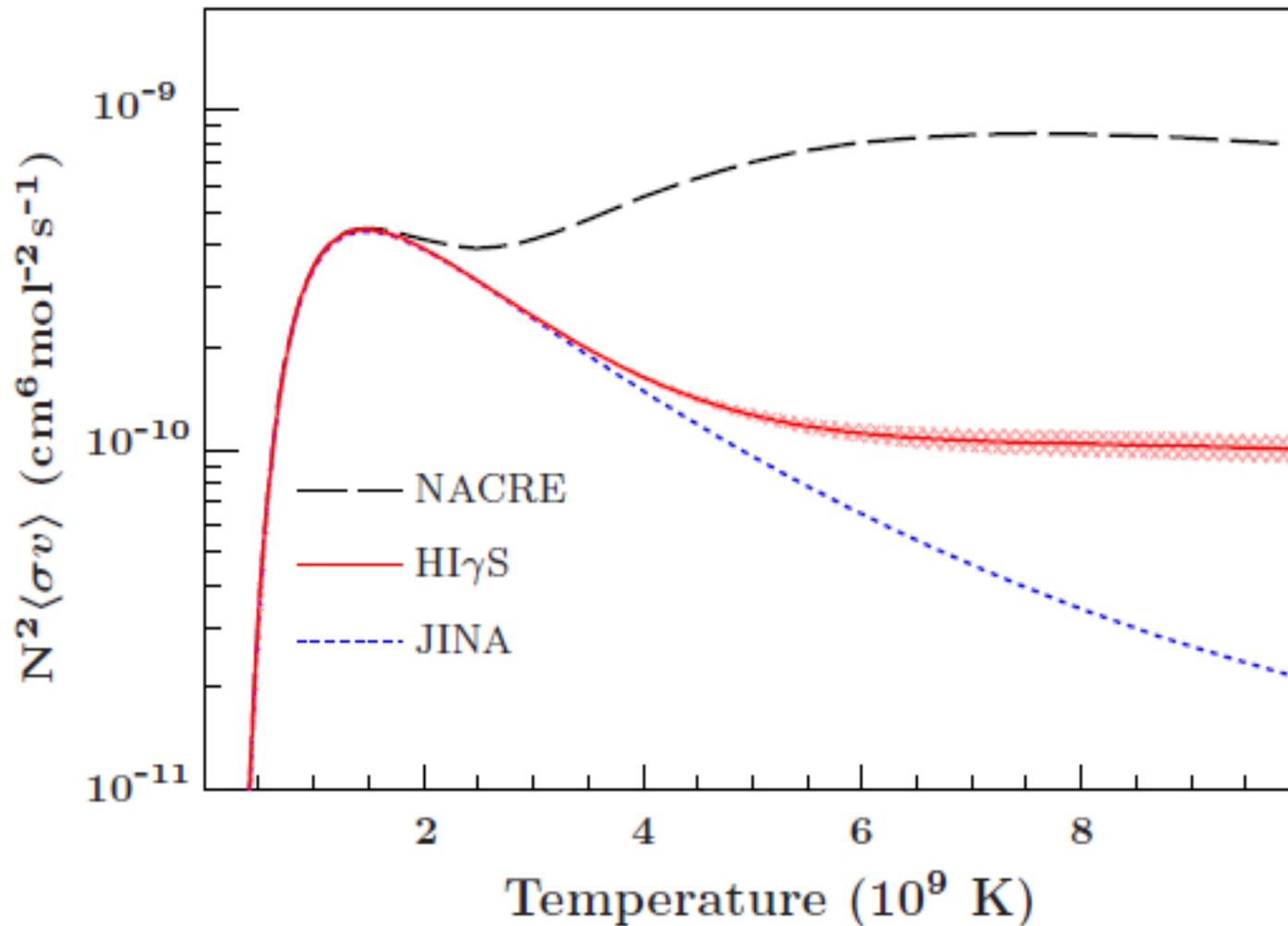
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# Triple- $\alpha$ Reaction Rates

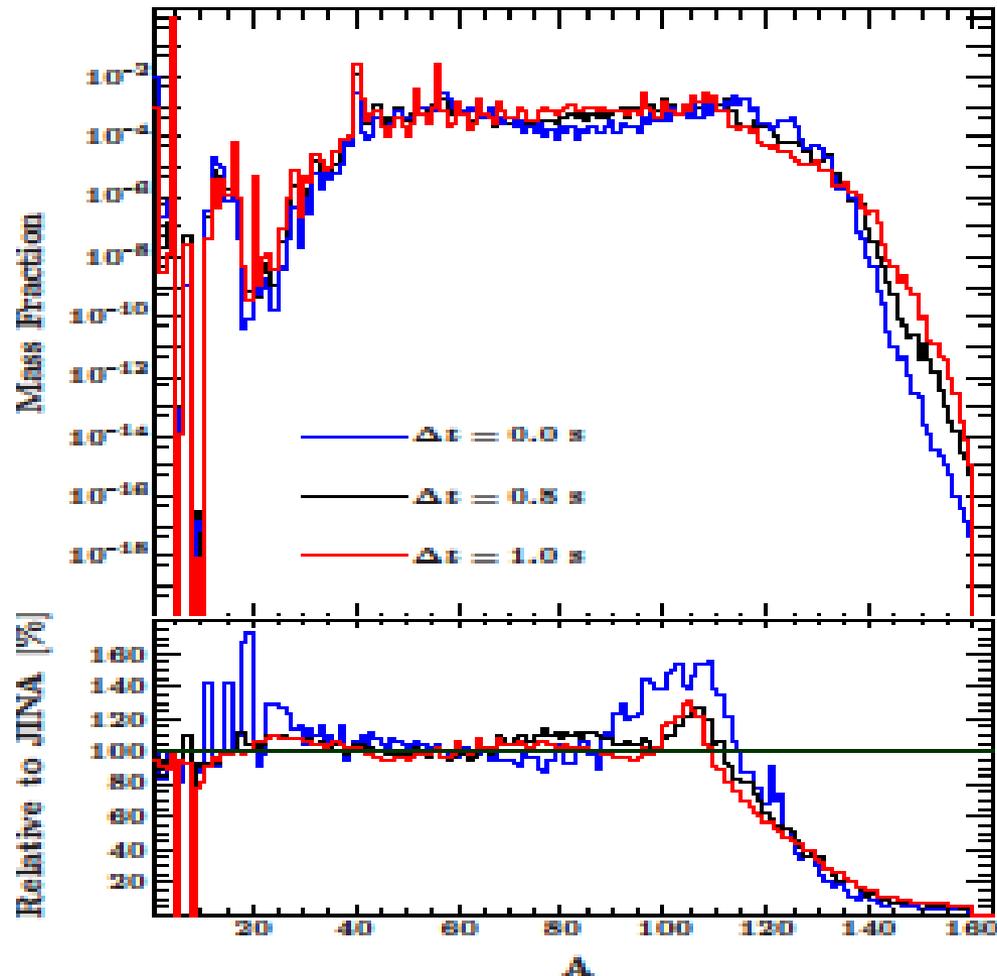


# Triple- $\alpha$ Reaction Rates



Carla Froehlich (NCSU)—Simulation of ejecta from supernova explosions based upon nucleosynthesis via the  $\nu p$  process. Explains the large abundance of Sr observed in metal poor stars. Our results (vs JINA) impact the production of elements having  $A > 90$ .

*Ref. Arcones, Froehlich and Martinez-Pinedo, ApJ 750:18 (2012)*



# Summary

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$E_{\text{res}}$ (MeV)	$\Gamma_{\alpha}(\text{res})$ (keV)	$\Gamma_{\gamma_0}(\text{res})$ (meV)	$B(E2 : 2_2^+ \rightarrow 0_1^+)$ ( $e^2\text{fm}^4$ )
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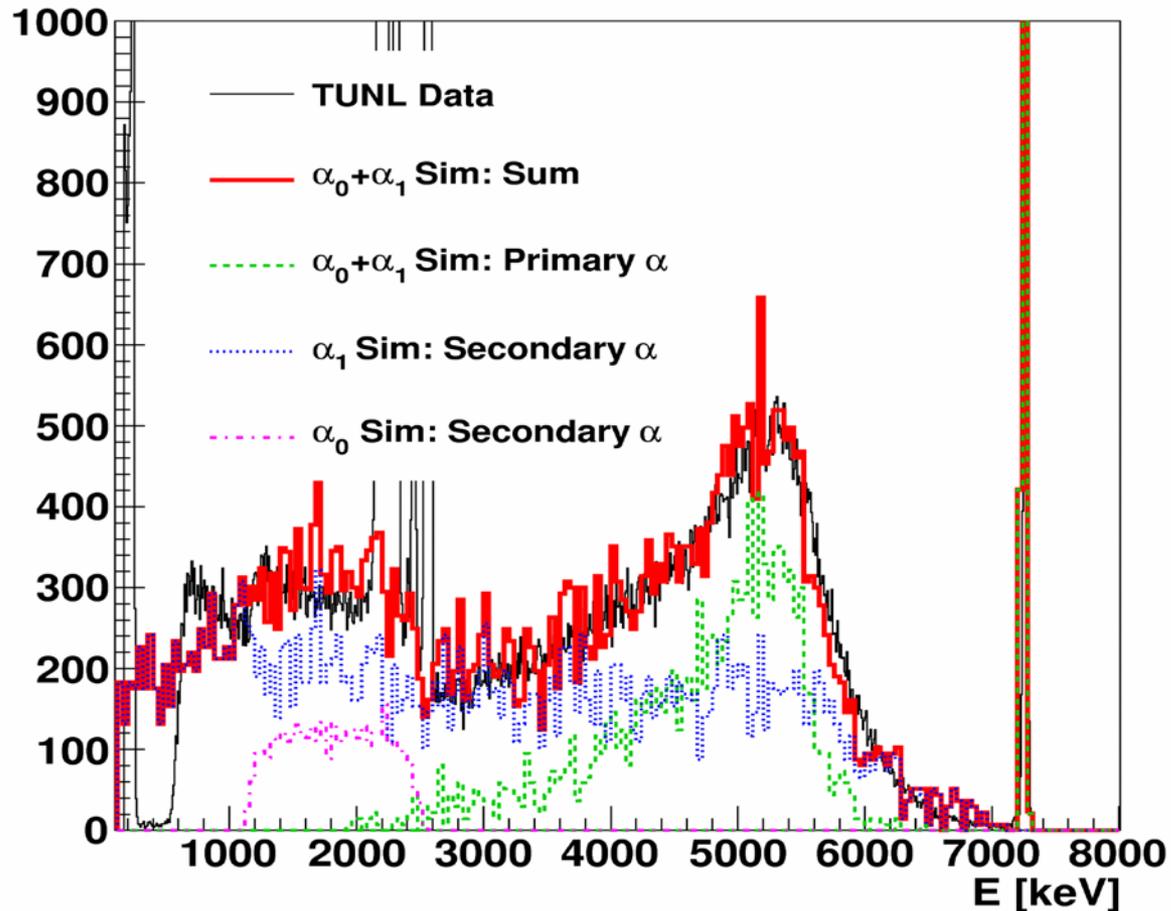
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- Revised triple- $\alpha$  reaction rates can affect nucleosynthesis of heavy elements during explosive astrophysics scenarios.

# Acknowledgments

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Sean Stave, and Henry Weller  
Triangle Universities Nuclear Laboratory

Moshe Gai  
Laboratory for Nuclear Science at Avery Point

*Success of the two step model at the 2.64 MeV  
3<sup>-</sup> resonance ( $\Gamma = 400$  keV).*



## **Cross Section for the $^{11}\text{B}(p,\alpha)^8\text{Be} \rightarrow \alpha\alpha$ Reaction**

**Although 3  $\alpha$ -particles are emitted, the number in a given energy interval depends upon the reaction dynamics.**

**Therefore, instead of cross section, we report  $X = \text{Counts/Luminosity}$**

$$\text{ie. } X = \frac{\text{Counts}}{N_t N_p dW} \text{ cm}^2/\text{sr}$$

**$N_t = \#$  of target nuclei/cm<sup>2</sup>**

**$N_p = \#$  of incident protons**

**$dW =$  detector solid angle**

**$X$  has the same units as differential cross section, but the # of  $\alpha$ -particles has not been divided out. It allows the user to find the number of outgoing alpha particles at any energy and angle where  $X$  is known for any specified Luminosity.**