Comment on “Staged z-pinch modeling of high and low atomic number liners compressing deuterium targets using parameters of the Z pulsed power facility“ [Phys. Plasmas 28, 112701 (2021)]

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August 2022

This comment and a response by Ruskov, Ney, and Rahman have been accepted for publication in the Physics of Plasmas. Publication is tentatively scheduled in August or September 2022.

This preprint is provided with the understanding that changes may be made prior to publication.

Comment on “Staged z-pinch modeling of high and low atomic number liners compressing deuterium targets using parameters of the Z pulsed power facility“ [Phys. Plasmas 28, 112701 (2021)]

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ABSTRACT

Analysis is presented that shows that the paper by Ruskov, Ney, and Rahman (Phys. Plas. 28, 112701, 2021) gives incorrect physical interpretations to liner-on-plasma computational results and has mathematical deficiencies in the computational approach. The analysis also confirms a previous result: Mach2 calculations of the so-called staged z-pinch artificially inject energy that leads to the incorrect conclusion that the calculations should reach fusion temperatures.

I. INTRODUCTION

Lindemuth, Weis, and Atchison[1] (LWA) conducted an independent review of Mach2 simulations of the so-called "staged z-pinch" (SZP), which is conceptually identical from a physics perspective to the generic liner-on-plasma that has been considered at least since Linhart[2]. LWA concluded that, in the parameter range considered, the SZP should not be considered to be a high-gain fusion concept. LWA reported three different, independent "stand alone" techniques to show that the previously reported Mach2 results exhibited non-physical behavior: (1) an analysis that the Mach2 calculations showed non-physical late time temperatures that were higher than could be obtained by post-shock adiabatic compression; (2) simple 0-dimensional modeling that showed fusion temperatures should not have been reached in the Mach2 calculations; (3) computed results obtained using three one-dimensional multi-material magneto-hydrodynamic (MHD) computer codes that did not reach fusion temperatures. The remarkable agreement between the three codes, which are based upon entirely different numerical techniques and boundary treatments, is particularly significant since the corresponding code experts worked more-or-less independently after problem definition and did not see results from other codes until right before publication.

Ruskov, Ney and Rahman[3] (RNR) attempted to discredit LWA by reporting a number of revised Mach2 calculations of the example designated as SZP2 by LWA. Surprisingly, seven different Mach2 modes of operation showed highly variable and significantly different computational results even though the exact same physical problem was addressed. The differences in results clearly could not be due to differing physical processes but, instead, must be due to how the different modes address the exact same physical processes. Nevertheless, RNR attempted to give physical, rather than mathematical, reasons for the differences.

RNR showed that two modes of operation of Mach2 could approximately match the LWA results, although the agreement by Mach2 was not as precise as the agreement between the three codes used by LWA. The two modes apparently used the same electrical circuit current as LWA. However, the actual current driving the pinch, as determined from Amperes law and the magnetic field at the liner outer boundary was 10-25% lower than the electrical circuit current. As discussed in Lindemuth's subsequent comment[4] (LC1), the actual current derived from Amperes law at the liner boundary defines the MHD problem. Of course, the circuit current and the Ampere's-law current should be identical, but for unexplained reasons, the two differed significantly in the two Mach2 calculations that approximately matched the LWA results. This is evidently a property specific to Mach2.

LC1 noted that the Raven liner boundary current as determined by Amperes law is, by computational design, the same value as the electrical circuit current. LC1 also noted that MHRDR's unique boundary condition is intended to ensure the full electrical current is carried in the liner, not the vacuum. Because of the different boundary approaches, the agreement of the three LWA codes up until late time is indicative that the boundary problem that evidently plagued the two Mach2 calculations simply was not present in the LWA calculations. Nevertheless, the approximate agreement of the two Raven calculations with LWA lead RNR to imply that the LWA codes must behave similar to Mach2 and that, therefore, the LWA calculations had a "likely problem" due to "incorrect treatment of the liner/vacuum boundary."

LC1 reviewed RNR and pointed out the physical inconsistencies of the Mach2 results. Most importantly, LC1 showed that Mach2 clearly had an energy conservation issue. In particular, LC1 formulated another "stand alone" technique: an analytic estimate that showed that during the initial shock launch and reflection process, a non-physical amount of energy was being injected into the calculations. This, of course, explained why the two RNR calculations that had an Ampere's-law current that was only 75-90% of the electrical circuit current, could approximately match the LWA results, where the Ampere's-law current was equal to the electrical circuit, and why Mach2 calculations with an Ampere's-law current equal to the electrical circuit current could reach non-physical fusion temperatures.

Ruskov, Ney, and Rahman[5] (RNRC1) did not provide any specific physics-based counterarguments (e.g., Rankine-Hugoniot conditions, LC1 Eq. 2, etc.) to the LC1 energy conservation conclusion but implied, without basis, that such analysis was wrong. Once again, RNRC1 also insisted "the three LWA codes were incorrectly set up" without acknowledging that the LWA code results were consistent with the LC1 analytic upper bound estimate.

In spite of the LC1 conclusions, Ruskov, Ney and Rahman[6] (RNR2) have published new calculations in the same parameter range as SZP2, including one using a silver liner that is identical in density and dimension to SZP2 and driven by a very similar electrical circuit current. In this Comment we will discuss errors in the RNR2 computational interpretation and in the computational approach and we will show that the new calculations amplify the LC1 conclusions.

We will also briefly discuss more accurate MHRDR calculations of the same problems. Whereas LWA MHRDR calculations used a vacuum region, calculations reported here use MHRDR in a mode where the electrical circuit current is applied directly to the liner boundary, i.e., the circuit current and the Ampere's-law current are identical. The arguments of RNR claiming LWA boundary issues simply do not apply here (and, as discussed previously, to Raven).

It is emphasized that the computational accuracy issues discussed in LWA, LC1, and this Comment are mathematical and part of the code verification process that determines whether or not "a particular computer code yields accurate solutions of the underlying analytical model[7]." Experimental data and the question of whether or not Mach-2 calculations can match experimental data in some regimes are not relevant.

II. LINER-ON-PLASMA PHYSICS

Of course, the purpose of numerical simulation of plasma physics concepts is to provide insight into the interplay of the physical processes involved in the underlying analytical model. Calculations must be first mathematically accurate, then correctly interpreted from a physics perspective. Even assuming the RNR2 calculations are at least qualitatively correct, the physical interpretations provided by RNR2 are incorrect.

RNR2 has reported calculations using tantalum, silver, and beryllium liners. The silver liner parameters are identical and the drive current is similar to that used for SZP2, so the silver liner should perform very similarly to SZP2 and, presumably, the tantalum and beryllium liners should also have qualitatively similar behavior.

In their conclusion, RNR2 notes that the magnetic "field builds up at the liner-target interface which launches magneto-sonic shocks into the target plasma." However, all of the previously reported Mach2 calculations, including RNR2 Fig. 13 (as discussed later in this Comment), show that the magnetic pressure is significantly lower than the fuel pressure. Therefore, the dominant fuel heating mechanism is hydrodynamic heating by the liner and the role of the magnetic field in the fuel can, for the most part, be ignored. In general, any shocks in the target plasma are launched by the momentum of the liner, not the magnetic field. They are not magnetosonic, they are hydrodynamic.[8]

The Rankine-Hugoniot conditions show that, in a strong shock situation, the velocity of the piston, and no other parameter, determines the properties of the shocked material. Furthermore, the rate of post-shock hydrodynamic heating done by the piston is determined by the velocity. As RNRC1 correctly notes, "proper insight into the pinch dynamics, shock wave propagation, and heating" requires "analyzing a multitude of plasma profiles." Therefore, considering that the liner velocity is the most important physical quantity for shock generation and hydrodynamic heating, it is remarkable that absolutely no radial velocity profiles are provided in RNR2 and that not even one value of velocity is mentioned for any of the three RNR2 liners. In addition, there are no profiles of liner/plasma interface radius as a function of time, so it is difficult to correlate liner position with the features in many of the RNR2 time plots.

RNR2 says there are "shocks," i.e., more than one shock, in the implosions. As previously noted, the new silver liner calculation should show behavior quite similar to SZP2. LWA showed, and presumably an analysis of Mach2 radial profiles would show, that there is exactly one shock in SZP2. The shock is launched when the liner "jumps off." The shock reflects off the axis and then moves outward. By the time the shock returns to the liner, non-adiabatic heating ceases and the subsequent heating of the fuel is quasi-adiabatic (e.g., see LWA Fig. 7 and note how the mass averaged temperature begins a second non-adiabatic increase just as the inward shock reaches the axis at $C_R \sim 3$).

Shocks are caused by a pressure discontinuity or a sudden increase in compressional velocity and are indicated in radial profiles by discontinuities in both pressure and velocity. RNR2 presents one pressure profile that shows a single pressure discontinuity, i.e., a single shock (RNR2 Fig. 7). RNR2 uses two <u>local</u> parameters, a so-called "Mach number" and a so-called "ram pressure," in an attempt to explain the shock dynamics. As used by RNR2, these parameters are essentially meaningless and are not the cause of shocks, but, rather, the result of a shock passing through the material.

RNR2 states that "a third of the plasma column ... is supersonic ... which generates strong shocks." What RNR2 calls a "Mach number" is the ratio of the local magnetoacoustic (essentially the sound) speed to the fluid velocity <u>at exactly the same location in the shocked material</u>. Of course, true Mach numbers are generally computed by comparing the velocity of a piston, e.g., an airplane or a liner, to the sound speed of the material <u>in front of the piston</u>, i.e., the Mach number is the ratio of quantities from two different locations.

For the cases considered in RNR2, a Mach 3-4 shock is being driven into the initially 2 eV fuel. Note that the "Mach numbers" in RNR2 Fig. 7 are significantly lower than the true Mach number. For a strong cartesian shock (Mach number much greater than unity), the Rankine-Hugoniot conditions show that the post-shock local "Mach number" is slightly less than unity, and is a result of the shock, not the cause of the shock. Because liners-on-plasmas are convergent geometries, subsequent adiabatic heating of the shocked material can increase the sound speed.

Again, what RNR2 calls a Mach number is a result of a shock, not the cause. A local "Mach number" much greater than unity does not mean strong shocks. As an example, consider the local "Mach number" of the air in the International Space Station: the local velocity is much higher than the local sound speed, and yet no shocks are generated.

RNR2 Fig. 7 defines the "ram pressure" as the quantity ρv^2, where ρ is the mass density and v is the local velocity. In the Eulerian form of the MHD equations, the quantity ρv^2 is the convection (often called advection) of momentum, i.e., it represents the momentum that is carried along with the movement of mass. Again, using the cartesian Rankine-Hugoniot conditions for an ideal gas, the post-shock value of ρv^2 is twice the kinetic energy density and 3 times larger than the pressure as a result of the shock, not as the cause of the shock.

In the Lagrangian formulation of the MHD equations, there are no convection terms, i.e., the "ram pressure" does not even appear in the equations. Most importantly, for both Lagrangian and Eulerian formulations, the term ρv^2, just as the mass convective term ρv, does not appear in the surface integral that defines the rate of change of the total energy within the fuel volume because there is no mass or momentum convective flow across that surface. The concept of ram pressure is generally invoked in steady-state situations, i.e., $\frac{\partial}{\partial x}(p + \rho v^2) = 0$, where p is the material pressure, so that the pressure where v=0 is the "ram pressure" where p is zero, again not a local concept but, instead, involving two different locations.

We can compare the "ram pressure" values in RNR2 Fig. 10 with estimates of the peak pressure at stagnation that can be obtained from Fig. 6. For Be, we get a fuel temperature of 300 eV and density ρ=5.4 x 10^3 kg/m^3 at C_R=30, where C_R is the convergence ratio R_o/R and R_o and R are the initial and present liner radii, respectively. Hence, using the ideal gas law for estimation purposes, the fuel in the Be target reaches a peak pressure of 1.1 x 10^{14} Pa. Corresponding values for Ag and Ta are 2 keV, 2.2 x 10^4 kg/m^3, 60, 4.1 x 10^{15} Pa, and 4 kev, 2.4 x 10^5 kg/m^3, 200, 9.2 x 10^{16} Pa. Comparing these pressure values with the peak "ram pressure" values in Fig. 10 shows that the beryllium peak pressure is more than an order of magnitude higher than the corresponding Fig. 10 value, silver nearly an order of magnitude, and tantalum 70% higher. This comparison demonstrates the essential irrelevance of the so-called "ram pressure" to a discussion of liner physics.

The discussion accompanying RNR2 Fig. 11 examines "pressure differentials." Of course, there must be a differential of the "ram pressure" at the liner/target interface because this is a contact discontinuity where the liner density is in general higher than the target density. Because the velocity is continuous after the shock passes the interface, ΔP_{RAM} is essentially $v^2\Delta\rho$, where $\Delta\rho$ is the change in density. There is no physical significance to this quantity.

From the equation of motion, we know that liner deceleration and turn-around is caused by the negative pressure gradient that occurs when the pressure inside the fuel builds up due to the hydrodynamic heating. It is perhaps surprising that tantalum oscillations occur when $-\Delta P_{TH}$ in RNR2 Fig. 11 is always a factor of 2 higher than ΔP_{MAG} and the two quantities are in phase

The RNR2 text notes that the peak magnetic field values at the liner/target interface are 50% higher than shown in RNR2 Fig. 13, i.e., the peak values for Be, Ag, and Ta are 2.1 kT, 23 kT, and 128 kT, respectively, and the corresponding magnetic pressures are 1.2 x 10^{12} Pa, 2 x 10^{14} Pa, and 6.5 x 10^{15} Pa. Although very high pressures, these magnetic pressure values are more than an order of magnitude lower than the peak fuel pressure as estimated in a previous paragraph. This indicates that it is the momentum of the liner that is driving the implosion and hence, as previously noted, the dominant heating mechanism is compressional heating. As the liner decelerates the kinetic energy of the liner is converted to thermal energy.

RNR2 does not discuss the material properties of the three liners and their role in the implosion process, instead focusing incorrectly on the "ram pressure." At any given temperature and density, higher-Z liners are at a lower pressure than lower-Z liners, i.e., the higher-Z liners are more compressible. Hence, higher-Z liners can be compressed more than lower-Z liners, leading the higher-Z liners during acceleration and implosion to behave much closer to point-mass kinetic calculations.

At the risk of oversimplification, the energy coupled to an imploding load in an inductive store circuit can be estimated as $0.5*\Delta L*I_{max}^2$, where I_{max} is the maximum load current, ΔL is the change in inductance per unit length of the load, i.e., $\Delta L = \frac{\mu_o}{2\pi}\ln C_L$, where C_L is the outer liner convergence ratio.

The approximate liner outer radius can presumably be estimated from RNR2 Fig. 13 as the point where the magnetic field profile begins to decrease approximately inversely as the radius. This leads to C_L values of approximately 100, 23, and 6.7, for Ta, Ag, and Be, leading to total load-coupled (liner + fuel) energy (thermal + magnetic + kinetic + radiation) estimates of 1.8 MJ/cm, 1.3 MJ/cm, and 0.76 MJ/cm, respectively. The higher compressibility of Ta and Ag, and the corresponding higher change in inductance, are reflected in the current waveforms of RNR2 Fig. 3. It is the higher coupled energy and the concurrent increased liner kinetic energy and momentum that lead to greater performance for the higher-Z liners.

Based upon "Mach2 simulations with all radiation turned off," in which "the inner surface of the liner did not radiate and heat the target plasma," RNR2 implies that radiation from the liner is a significant fuel heating mechanism. In general, certainly at late times, the fuel is at a higher temperature than the liner, and the fuel heats the liner, rather than vise versa. However, a MHRDR Ag simulation shows that early in the implosion, from approximately 76 ns to 110 ns, i.e., before C_R=2, approximately 100 J/cm of radiation from the liner heats the fuel. During this same period, the compressional heating is 1.3 kJ/cm, so radiative heating of the fuel is a minor effect. In contrast, the Be liner is always colder than the fuel.

RNR2 claims the magnetic field "suppresses the electron heat loss across the interface" although this statement is another one not supported quantitatively. Approximate fuel parameters from Figures 6 and 13 at peak compression can be used to estimate $(\omega\tau)_e$, the electron cyclotron-frequency/collision time parameter. Although this parameter may be marginally higher than unity at peak compression, it is proportional to $(C_R)^2$, so that it is generally less than unity. Therefore, the electron heat loss is not magnetically suppressed, i.e., the RNR2 targets are not magnetized targets. As LWA noted, in the SZP high-fuel-density range, radiation losses dominate so magnetic reduction of electron thermal conduction would not play a major role.

Similarly, without quantitative support, RNR2 makes the hypothesis that, at late time, "magnetosonic radiative shocks, by transporting mass to the shock front, build up a dense silver or tantalum layer at the liner-target interface." Of course, examination of radial profiles would show that there are no shocks once the initial shock is reflected off the axis. Furthermore, shocks do not "transport" mass, they compress material. And, as shown in the examination of magnetic field pressures, the implosion of the fuel is not magnetosonic, it is essentially hydrodynamic.

RNR2 has significantly misinterpreted the physical behavior of liners-on-plasma. While this section has not been a complete study of liner-on-plasma physics due to publication space limitations, it has been an attempt to correct many of the RNR2 misconceptions.

III. COMPUTATIONAL DEFICIENCIES

Computational techniques can affect both the accuracy of a numerical simulation and the subsequent interpretation. For one-dimensional compression problems such as liners-on-plasma, a Lagrangian code is generally considered the appropriate choice. Although one RNR Lagrangian calculation gave high-gain results that RNR put forward as an accurate answer, RNR2 instead used a "pure Eulerian" formalism even though RNRC1 noted that the RNR Eulerian calculation "second shock is weaker ... 60 eV ...vs. 150 eV." In contrast to Lagrangian calculations, Eulerian cells can have more than one material in them. The treatment of mixed cells is clearly a potential source of error that would not be present in Lagrangian simulations.

When the grid remains fixed in space, the number of cells in the fuel significantly decreases just when *a priori* one would think finer resolution would be required. The Ag liner in RNR2 reaches approximately C_R=60, i.e., a radius of 33.3 μm, so, using RNR2 Table I, the fuel is resolved by only 21 computational cells at peak compression. Similarly, the Ta liner reaches C_R=200, which leads to a fuel resolution of only 13 cells. Both resolution values would *a priori* be considered inadequate, although this may be adequate for quasi-uniform radial profiles. RNR2 does not report any grid convergence studies, but RNR2 does note that "this particular grid definition was informed by arbitrary Lagrangian-Eulerian simulations," which were not reported.

In discussing Fig. 6, RNR2 notes "for C_R values between approximately 3 and 5, two additional shock preheating intervals for Ag and Ta are visible." None of the LWA codes showed such a double heating behavior. Although RNR2 shows no density and velocity radial profiles in the C_R=3-5 range, the LWA calculations of SZP2 and MHRDR calculations using the RNR2 silver liner/fuel parameters show very clearly that the non-adiabatic heating is due to the reflection of the shock off the axis and there is only one continuous, additional "interval," not "two additional shock preheating intervals." We note that as the liner moves from C_R=3 to C_R=5, the liner encounters a discontinuous change in grid cell sizes (see RNR2, Table I). It seems reasonable to conclude that the "two" mass-averaged temperature behavior is a grid size effect and not physical.

MHD codes such as Mach2 solve initial-value/boundary-value problems. As discussed in LC1, the boundary value, i.e., the Ampere's-law current, should be the external electrical current. Regarding initial liner conditions, RNR2 notes "the transitioning from a solid liner to a liner plasma is a complicated process, and we do not have access to codes which can model it." Therefore, RNR2 used an expanded liner at a density of 600 kg/m^3, less than 1/3 the Be solid value and less than 6% of the Ag solid value, and at 2 eV, instead of a liner at solid density and standard temperature. The Sesame library equation-of-state (EOS) 2023 gives a Beryllium pressure of 2.2 x 10^{10} Pa for this initial density and temperature and EOS 2720 gives a silver pressure of 1.9 x 10^9 Pa.[9] The beryllium value is comparable to the pressure that *a priori* might be expected the fuel would reach only after an initial shock, e.g., LC1 estimated the SZP2 shock pressure to be 4.7 x 10^{10} Pa. Therefore, it would be expected that the beryllium simulation would exhibit a strong liner expansion even before significant electrical current flows in the liner. The expanding liner would be expected to have a significant impact on the fuel. Even the silver liner would be expected to have some pre-shock influence on the fuel. RNR2 Fig. 6 apparently shows this effect, where the Be target shows some early adiabatic heating prior to the beginning of non-adiabatic heating.

Corresponding MHRDR "cold-start" computations using the same liner and fuel mass as RNR2 have been conducted for the Be and Ag liners initially at solid density and 0.03 eV. The liner center-of-mass is located as in RNR2, i.e., 2.55 mm. The fuel is also at 0.03 eV and, since the target initial radius is larger, the density is correspondingly reduced to maintain the same mass as RNR2.

The MHRDR "cold-start" calculations are, of course, more realistic than the RNR2 calculations, and they give significantly lower temperatures than the RNR2 results and even lower than the LWA SZP2 results because the shock must also provide the dissociation and ionization energy of the fuel. In particular, the MHRDR cold-start Ag calculations reach a maximum mass-averaged temperature of about 600 eV at C_R=100 and the Be calculations 300 eV at C_R=50. A typical MHRDR calculation takes less than 15 seconds on a 2012 MacBook Pro. The MHRDR "cold-start" results reinforce the LWA conclusion that the staged z-pinch is not a high-gain fusion concept. Publication space limits preclude the presentation of a detailed analysis of the MHRDR calculations.

IV. THE ENERGY CONSERVATION ISSUE

Using reasonable assumptions and the Rankine-Hugoniot conditions for a strong cartesian shock, LC1 derived an expression for an upper bound on the post-shock work that could be done on a fusion target by an imploding liner:

$$\frac{W}{L} = \frac{6\pi P_o R_o^2}{C_j^2}\left(1 - \frac{1}{({}^{C_R}/_{C_j})^{1/3}}\right) \qquad (1)$$

where, as in LC1 Eq. 2, W is the work done by the liner, L is the length of the liner, R_o is the initial liner/fuel interface position, P_o is the post-shock pressure in the fuel immediately after the shock passes, $C_R=R_o/R$ is the usual convergence ratio, where R is the position of the liner, and, introducing a new notation, $C_j=R_o/R_j$, where R_j is the "jump off" radius of the liner when the liner shock starts in the fuel. Normally, $R_j=R_o$ and C_j=1, but to correctly analyze an RNR calculation, LC1 used C_j=1.3 since the liner had moved significantly inward prior to shock "jump off."

As discussed in LC1, Eq. 1 is applicable until the shock driven by the liner piston is reflected off the axis and returns to the liner. The work done by the liner appears as thermal and kinetic energy in the fuel, but when the shock returns to the liner, all of the work done up to that time appears essentially as fuel thermal energy. Therefore, Eq. (1) can be used to establish an upper bound on the maximum average temperature T_{max} that can be obtained when the shock returns to the liner

$$T_{max} = \frac{8 m_i v^2}{3 k_B C_j^2}\left(1 - \frac{1}{({}^{C_R}/_{C_j})^{1/3}}\right) \qquad (2)$$

where m_i is the ion mass, v is the liner velocity, and k_B is Boltzmann's constant.

RNR2 has reported calculations using tantalum, silver, and beryllium liners. For the silver liner, RNR2 Fig. 6 shows a mass-averaged temperature of 100 eV at C_R=5. The silver liner parameters and the drive current are similar to that used for SZP2. Therefore, the "jump off" velocity should be approximately the same, i.e., 6 cm/μs (as remarked earlier, RNR2 does not provide any information regarding liner velocity). Since, as noted in the previous section, some initial liner movement prior to shock break out would be expected due to the high liner initial pressure, a C_j greater than unity is probably appropriate. However, even for C_j=1, Eq. 2 indicates an upper bound of 83 eV at C_R=5. Therefore, the RNR2 silver calculation exhibits the same artificial injection of energy as RNR.

For reference, a corresponding MHRDR calculation using the RNR2 initial conditions shows the shock returning to the liner at C_R=4, where the mass-averaged temperature is only 50 eV, which, as expected, is below the Eq. 2 upper bound value of 74. The MHRDR calculations ultimately reach a maximum temperature of 800 eV at a convergence of 90. The peak temperature is significantly higher than the MHRDR cold-start value, although not in the fusion range obtained by the RNR2 Mach2 calculations.

V. CONCLUSION

We have shown that the RNR2 physical interpretation of their Mach2 computational results is incorrect, and we have corrected those errors. We have also shown that the computational approach has some deficiencies, in particular, with zoning and initial conditions.

RNR2 are qualitatively correct that a high-Z liner gives better performance than low-Z liners at least in one-dimensional MHD simulations, as is generally recognized throughout the liner implosion community. However, once again, analysis of published Mach2 calculations of liners-on-plasma confirms the LWA conclusion: the calculations should not have reached fusion temperatures.

From a fusion energy perspective, we have essentially a "go/no-go" question: should Mach2 calculations using the proposed staged z-pinch parameters reach fusion temperatures, as RNR and RNR2 continue to insist, or should fusion temperatures be unobtainable, as demonstrated in LWA, LC1 and here. Of course, regardless of fusion relevance or lack thereof, liners-on-plasma experiments and calculations provide a platform for very interesting plasma physics. Both experimental and computational data must be properly interpreted.

APPENDIX: THE RNR2 RESPONSE

The authors evidently still do not recognize that they have not explained why the circuit current and Ampere's-law current differ so significantly in two RNR Mach2 modes; simply showing that other modes make the two currents approximately equal is not an explanation. They, of course, have claimed that the Mach2 modes where the two currents are approximately equal give accurate results, but they reject calculations where the two currents are not just approximately equal but are precisely equal (LWA Raven calculations and the MHRDR calculations discussed in this Comment). The authors continue to "believe that the LWA calculations do not evolve the magnetic field correctly."

In their response, the RNR2 authors state "we disagree with the author that there is only one shock launched when the liner "jumps off"" and they continue to allege that there are "secondary shocks." To support these claims, the authors offer Figures 1 and 2 of the response. In fact, these Figures actually confirm aspects of the discussions in LWA, LC1, and this Comment.

Fig. 1, left, shows velocity and pressure profiles between C_R~2.9 and C_R~4.2. Labeled "the first shock heating interval," this is evidently the same as the first of what RNR2 called the "two additional shock preheating intervals" of RNR2 Fig. 6 that occur "for C_R values, between approximately 3 and 5." (in RNR2 Fig. 6, the first shock preheating occurs before C_R~3). Fig. 1, left, confirms that the second of the two additional intervals, which begins at C_R~4 (RNR2 Fig. 6), begins when the liner crosses the zoning discontinuity at 0.5 mm (RNR2 Table I).

There is, in fact, a weak precursor wave in Fig. 1, left, apparently due to the high pressure that corresponds to the initial conditions (RNR2 notes that there is a "slow pinch compression … when the target shrinks to 50% of its initial radius"). With the exception of the precursor, there is one strong shock, not multiple "strong shocks" (RNR2 abstract), in each radial plot. The reflected shock is the reflection from the axis of the single shock launched when the liner jumps off.

Fig. 1, right, shows profiles between C_R~5.1 and C_R~5.7. Labeled "the second shock heating interval," this is not the same as the second of what RNR2 called two additional intervals of RNR2 Fig. 6, which ends, according to the RNR2 text, at C_R~5. Curiously, there are no plots in Fig. 1 between C_R~4.2 and C_R~5.1, the period in which the liner crosses the zoning discontinuity and the period in which the outward moving shock reflects off the liner.

Fig. 1, right, is alleged to show shocks and is evidently what the authors refer to as a "secondary" shock. This obviously contradicts RNR2 Fig. 6 that shows that "the enhanced shock preheating" ceases beyond C_R~5. Fig. 1, right, shows an inward wave moving at 10-12 cm/μs. RNR2 Fig. 6 indicates that, for the silver liner, the fuel at C_R~5 has a temperature of about 100 eV, i.e., a sound speed of ~ 12 cm/μs. The inward wave of Fig. 1, right, is therefore moving at approximately the sound speed. Hence, the inward moving wave shown in Fig. 1, right, is a large amplitude sound wave, not a strong shock wave, and, as shown in RNR2 Fig. 6, produces little or no additional entropy. There is no indication in Fig. 1 that there is any wave motion due to anything other than the initial shock.

Figures 1 and 2 of the response qualitatively confirm the process of an initial, single shock launched by the liner reflecting off the axis and the liner and dissipating into a sound wave as discussed in LWA Fig. 7 and Table IX.

In the analysis associated with Equations 1 and 2 of this Comment, a liner velocity of 6 cm/μs was used, leading to an Eq. 2 C_R=5 fuel temperature upper bound of 85 eV. However, a liner velocity of 5.3 cm/μs can be inferred from the response Fig. 1, left. This leads to a more accurate upper bound of 66 eV, a value far exceeded by the RNR2 Ag calculation, again confirming a very significant artifical injection of energy. The response insists that the "main argument ... is ... three different codes ..." In fact, the more convincing argument should be Eq. 2 of LC1 and Equations 1 and 2 of this Comment.

Regarding the so-called "ram pressure," the response states "obviously, it transfers momentum from the liner side of the liner/fuel interface, to the fuel." This statement is patently false. As noted in the main text of this Comment, the term ρv^2 is a convective term that represents momentum carried along with the mass flux ρv. There is absolutely no mass, and hence absolutely no momentum, convected from the liner side of the interface to the fuel. This can be demonstrated easily by integrating the equation of motion over the fuel outer surface.

The response notes that "the Sesame thermal conductivity does not distinguish between heat transport in the parallel and perpendicular direction of the local magnetic field." Presumably, this indicates that the magnetic reduction of thermal conductivity was not even included in the RNR2 calculations. Hence, there is absolutely no basis for the RNR2 statement that the magnetic field "suppresses the electron heat loss across the interface."

The response offers calculations using a reduced "initial plasma temperature of 0.025 eV" to refute the MHRDR "cold-start" results. Presumably, these calculations did not use solid-density liners at the lower temperature, although this is not clearly indicated. Nevertheless, such calculations would be expected to have the same artificial energy injection issues as all of the other Mach2 calculations.

Finally, the response offers three global energy plots that apparently indicate total system energetics, rather than fuel-specific parameters, and claims "the sum of these four energy components ... is smaller, or at most equal to the circuit E_{cir} energy driving the system, indicating that the energy in the simulation is conserved". In the context used in LC1 and this Comment, energy conservation means a comparison of the energy in the fuel to the energy that comes across the fuel outer surface. The statement that "the sum ... is smaller" is certainly not a proof of fuel energy conservation. Regardless of the interpretation of the energy plots, the magnitude of the non-physically injected energy, as determined from Eq. 1, will be of the order of 10 kJ/cm, depending on problem. The injected energy is a small fraction of the total energy available and a value very difficult to detect on the scale of the energy plots. And, as well known, even if a computational approach conserves energy exactly, the scheme could introduce non-physical exchanges between the various forms of energy (I. R. Lindemuth, J. Comput. Phys. 18, 119 [1975]; erratum, J. Comput. Phys. 19, 338 [1975]).

The energy plots simply do not disprove the conclusions based upon Eqs. 1 and 2 (and LC1 Eq. 2). As with RNRC1, the response makes no specific physics-based counterarguments to the derivation of these equations.

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[7] See https://aip.scitation.org/php/info/policies for definitions of code verification and code validation.

[8] LWA Fig. 11 and discussion show that under some circumstances field diffusion prior to liner shock breakout can drive a z-pinch into the fuel, but the compression by the liner remains the dominant heating mechanism.

[9] RNR2 does not specify which Sesame libraries are used to obtain the EOS, transport coefficients, and opacities required for their calculations. As noted by LWA, early Mach2 calculations apparently used dysprosium opacities for silver. We know that silver and tantalum opacities are not available in public Sesame libraries.